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Sales Prediction in the Ice Category Applying Fuzzy Sets Theory

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Abstract

With growing pressure on performance and data regarding customer behaviour and supply chain process widely available, stock keeping units aim to optimise the level of inventories. It is natural that good estimates of future sales can substantially increase the efficiency of the overall company. We can distinguish two basic perspectives: one assumes sales to be an independent process; the other explores its dependency on exogenous variables. In this paper we focus on the forecasting of sales in the Ice category when dependency on quarterly average temperatures in the form of exponential function is assumed. We concentrate especially on LFL-Forecaster, a method combining fuzzy transform and fuzzy natural logic of fuzzy sets theory, as a tool for average temperature forecasting. The results are compared with simple linear extrapolation and truly observed temperatures. The utilisation of LFL-Forecaster is found to be superior to simplifying linear regression.

Keywords

Forecasting, fuzzy sets, ice category, sales, temperature.

JEL Classification: C44, C61, G22

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1. Introduction

Reliable estimation of customer demand for products and services constitutes a key aspect of financial planning in every company, but especially in the retail sector owing to the huge transfer of information, materials, products and funds among all parts of the supply chain (see, e.g., Syntetos et al., 2016, for a comprehensive discussion of supply chain forecasting). Given that obtaining direct data regarding customers' behaviour and its potential to change is not usually feasible, an indirect approach is often adopted, primarily relying upon past sales observations.

However, sales typically depend on various endogenous variables, such as GDP, unemployment or CPI, which cannot be planned easily, or product-related parameters (e.g., life period), which in contrast can be well-predicted. In addition to pure economic quantities, we can often find a good proxy of sales trends when variables specific to a given product are considered, such as natality for infant products and services or average temperature for leisure activities and related products.

Therefore, the basic notion is to use historical time series in order to identify a suitable variable or variables that allow one to forecast future sales as effectively as possible. The key assumption is that if some relation was valid in the past, it should remain relevant in the future. Of course, such a relation should be supported by well-established theory and not represent a result of pure statistics alone (see, e.g., Armstrong et al., 2015, for relevant discussion).

To the best of our knowledge, barely any previous research has sought to analyse the relationship between sales and (average) temperature, even though several authors have discussed customers' tendency to shop in various weather conditions (see also Davenport, 2014).

Nevertheless, Páník (2016) has recently studied the possible impact of quarterly temperature averages on sales magnitude for the Ice product category in a large conglomerate company in the Czech Republic, including the complex treatment of forecasted temperature averages on firm value and risk management.

In this paper we essentially use the same databases, but rather than using the standard estimation technique, we specifically concentrate on LFL-forecaster, which utilises the properties of fuzzy natural logic (see Novák, 2016).

We proceed as follows. The next Section briefly reviews some literature relevant to the forecasting of firm-specific variables, especially sales. In Section 3, the forecasting of (financial) variables is discussed, with special attention to the theoretical foundations of LFL-Forecaster. Section 4 then provides the data used in the empirical analysis and Section 5 displays the results. Section 6 concludes the paper.

2. Relevant literature review

With growing pressure on performance and data regarding customer behaviour and supply chain process widely available, stock keeping units aim to optimise the level of inventories, i.e., they mostly minimise stocks due to storage costs and tied-up capital. Moreover, customers' tastes can change suddenly and stocks may become unsellable. It is therefore natural that the accurate estimation of future sales can substantially increase the efficiency of the company as a whole.

A key question pertains to the horizon of forecasting to be considered, because different decisions require different horizons, while budgeting needs, aggregated annual data, distribution management and scheduling all require short-term data by location. Clearly, short-term forecasts should consider seasonal effects and may prove weather- (uncertain to happen) and holiday- (fixed) dependent, while long-term forecasts are rather economic cycle-dependent.

In this context, it is important to stress the quality and length of data used for the estimation of model parameters (especially crucial if new or re-designed products are introduced, the market is immature, customers' sentiment is unstable, etc.). Potential means of data translation have duly been studied, e.g., by Withycombe (1989), Chen and Boylan (2007, 2008) and Boylan (2010).

As concerns sales forecasting itself, we can distinguish two basic perspectives: one assumes sales to be an independent process, while the other examines its dependency on exogenous variables or so-called influencing factors (for an example of multivariate analysis, see, e.g., Guo et al., 2013). In the former, various kinds of autoregressive processes can be considered.

It is also interesting to study the relationship between particular stages of complex supply chains, i.e., the kind of demand propagation through the supply chain into the orders of, say, primary materials. For example, Lee et al. (2000) show that the bottom level can be forecasted quite well by the AR(1) process, while on top ARMA(1,1) should be preferred. The same authors argue that if there is positive autocorrelation of the demand, the variance of orders increases through the chain, which they term the *bullwhip effect*.

Another interesting question related to the forecasting of sales is the impact of abnormal events, such as promotional activities and sales of complements and substitutes, as discussed in Ma et al. (2016).

3. Forecasting of random variables

A sequence of random variables in time is referred to as a *time series*. Although in finance we usually assume continuous distributions, their observations are by nature discrete and thus a sequence in time is always finite.

In general, we can distinguish a trend part, L_t, a seasonal component, S_t, and a random noise, ε_t:

$$X_t = X_0 + L_t + S_t + \varepsilon_t, \qquad (1)$$

where X_0 stands for the initial value and the subscript *t* shows the dependency on time. While the trend part should depict long-term change in the random variable, the seasonal component should capture short periodically repeating changes, such as time of year/week/day. On the other hand, random noise is a kind of error between the predicted value and real observation, which cannot be explained by any function of time (its mean should be zero).

In the next lines we will briefly review a standard approach for linear forecasting of time series; subsequently, an advanced approach based on fuzzy-linguistic definitions will be discussed.

3.1 Standard approach

The simplest approach to the analysis of time series is to assume a linear relationship between dependent and independent variables, where the current value of X_t depends on its previous value (or values):

$$X_t = \beta_0 + \beta_1 X_{t-1} + \varepsilon_t, \qquad (2)$$

with regression coefficients (therefore linear regression) β_0 and β_1 usually obtained via either least-square etimation or maximum-likelihood estimation. Obviously, the error term ε should remain zero on average.

In case the error is too large or fluctuating, it might be useful to apply the regression on smoothed time series instead of using raw values. For example, X_{t-1} can be replaced by a simple moving average of *n* preceding values; in such a case particular error terms should partially be offset by themselves.

3.2 LFL-forecaster

The LFL-forecaster[1] combines the fuzzy transform (F-transform) technique to extract the trend part with fuzzy natural logic (FNL) in order to forecast future values, see Novák (2016). Both methods apply the principles of fuzzy sets, i.e., a function $A: U \to [0,1]$ with U being the universe and $[0,1]$ being its support set on standard algebra, to time series analysis.

F-transform

The principles of F-transform were formulated by Perfilieva (see, e.g., Perfilieva, 2006, as well as Holčapek and Tichý, 2011). The key idea is to transform a continuous function $f: [a,b] \to R$ to a finite vector of numbers (*direct F-transform*) and then transform it back (*inverse F-transform*) to the original space so that an approximating function $\hat{f}$ is obtained. The procedure is as follows.

First, form a *fuzzy partition* of a given domain $[a,b]$, which consists of a finite set of fuzzy sets $A = \{A_0, \dots, A_n\}$, $n \geq 2$, defined over nodes constructed on the domain, i.e., $a = a_0, \dots a_n = b$. The membership functions of $A_0, \dots, A_n$ into the partition A are called *basic functions*.

Having the partition A in place, we can define the direct F-transform as a vector $\mathbf{F}[f] = (F_0[f], \dots, F_n[f])$ with each *k*-th component $F_k[f]$ specified as follows:

$$F_k[f] = \frac{\int_a^b f(x) A_k(x) \mathrm{dx}}{\int_a^b A_k(x) \mathrm{dx}}, k = 0, \dots n. \qquad (3)$$

Such a partition is called h-uniform if the nodes $a_0, \dots a_n$ are h-equidistant, i.e., $a_{k+1} = a_k + h$ with $h = (b-a)n$ for all $k = 0, \dots, n-1$.

Subsequently, the inverse F-transform, which helps us attain an approximation of the original function, is a continuous function $\hat{f}: [a,b] \to R$ such that

$$\hat{f}(x) = \sum_{k=0}^{n} F_k[f] A_k(x), x \in [a,b]. \qquad (4)$$

Obviously, $\hat{f}$ should converge to f with $n \to \infty$.

More details about the F-transform as well as all related proofs can be found in Perfilieva (2006) and Perfilieva et al. (2011). Note also that the definition above is in fact, F^0-transform, i.e., a zero-degree F-transform, because the components are real numbers. Clearly, the

[1] Demo viz http://irafm.osu.cz/en/c110_lfl-forecaster/.

procedure can be easily generalised to F^m-transform, for example, with F^1-tranform the components are formed by polynomials, such as triangular or cosine functions.

Fuzzy natural logic

Following Novák (1995), fuzzy natural logic (FNL) is a group of mathematical theories that extends mathematical fuzzy logic in a narrow sense, thereby developing a mathematical model of special human reasoning when natural language is employed.

It is based on a set *EvExpr* of evaluative linguistic expressions. For our purposes, the most useful subclass of *EvExpr* is a class of *simple evaluative expressions* structured as:

$$\langle \text{linguistic hedge} \rangle \langle \text{TE adjective} \rangle. \tag{5}$$

Here, TE adjective forms fundamental evaluative trichotomy, with typical examples being *gradable adjectives* (such as *strong*, *long*, etc.) and *evaluative adjectives* (such as *low*, *medium*, *high*, etc.).

On the other hand, the so-called linguistic hedge represents adverbial modifications, such as *intensifying adverbs* of *narrowing sense* (such as *extremely*, *significantly*, etc.) or *widening sense* (such as *more or less*, *roughly*, etc.).

The idea behind its utilisation in time series analysis and forecasting is that FNL can allow us to transform past observations of the trend/cycle into the future, using natural language.

Trend estimation and its forecasting

Estimation of the trend/cycle part of the time series can be obtained following the theorem of Novák (2016) and Novák et al. (2014):

Theorem. *Let* X_t *be a realisation of random variable over interval* $[0, b]$*. Let us construct an h-uniform fuzzy partition A over nodes* $a_0, \dots a_n$ *with* $h = d\bar{T}$*, where* $\bar{T} = 2\pi/\lambda$ *for a minimal* $\lambda = min\{\lambda_1, \dots, \lambda_r\}$ *and a real number* $d \geq 1$*. If we compute a direct F-transform* $F[X]$*, then for the corresponding inverse F-transform* $\hat{X}$ *of X there is a certain small number D converging to 0 for* $d \to \infty$ *such that*

$$\left|\hat{X}_t - L_t\right| \leq D, t = |c_1, c_{n-1}|. \tag{6}$$

Such partition is called h-uniform if the nodes $a_0, \dots a_n$ *are h-equidistant, i.e.,* $a_{k+1} = a_k + h$ *with* $h = (b - a)n$ *for all* $k = 0, \dots, n - 1$.

As soon as the trend has been estimated, the new task is to extrapolate the known values of X (or rather $\hat{X}$) into l future intervals (a so-called forecasting) using the implications of fuzzy natural logic.

Given that direct F-transform leads to a vector of components,

$$\mathbf{F}[X] = (F_1[X], \dots, F_{n-1}[X]),$$

where each such component represent a weighted average of values of X_t in the area of width $2h$, they can be used as data to learn a linguistic description. Subsequently, using FNL, the future components can be forecasted,

$$F_n[X], \dots, F_{n+l}[X].$$

Finally, the inverse transform is used to calculate the trend/cycle development.

For more details of possible linguistic descriptions, see, e.g., Novák et al. (2010) and Novák (2016).

4. Data and methodology

Let us assume an adjusted data set of quarterly sales in the Ice category (various kinds of ice cream), over 2008–2014, i.e., 28 observations. The adjustment aims to offset stocking up (a partial intention of spring orders is to cover expected summer demand) by moving part of Q2 sales into Q3 and disguising firm-specific figures (the Ice category constitutes about one third of total sales) by multiplying them by an arbitrary number.

It would not be surprising if sales in the Ice category were strongly influenced by average temperature. Indeed, as documented in Figure 1 (for more details, see, Páník, 2016), simple exponential regression provides a quite good fit of quarterly sales on quarterly average temperatures: whereas winter months (Q1, Q4) are grouped in the bottom-left corner, summer months (Q2, Q3) are grouped in the top-right corner.

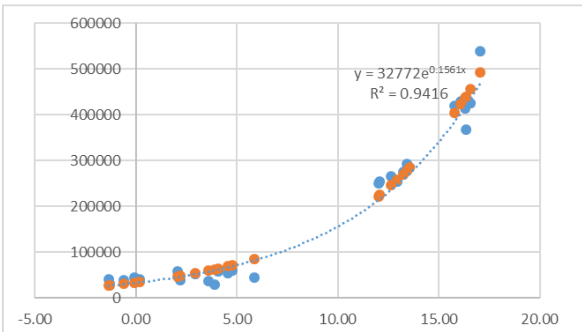

Figure 1 Quarterly sales on quarterly average temperatures (Páník, 2016)

It follows that having a good estimate of average temperatures should provide a clear picture regarding future sales. For this purpose, we collect average temperatures for each month in the Czech Republic[2] since 1961 and calculate quarterly averages. In the next section, we use these values to forecast future average temperatures and subsequently to calculate future sales, either by utilising a standard approach (linear regression for each quarter separately as in Páník, 2016) or applying LFL-Forecaster, which combines F-transform and FNL theory as described above.

5. Forecasting of Ice category sales

As explained above, the quarterly average temperatures over 1961–2014 are utilised to forecast future averages for the next six years by both standard (linear regression for each quarter separately) and alternative (LFL-Forestare) methods, and compared with real observations over 2015–2017 (see Figure 4).

Table 1 Forecasted values of quarterly average temperatures

Linear Regression (Páník, 2016)				
	Q1	Q2	Q3	Q4
2015	0.0735	11.9775	15.756	3.32
2016	0.0978	11.9976	15.771	3.3304
2017	0.1221	12.0177	15.786	3.3408
2018	0.1464	12.0378	15.801	3.3512
2019	0.1707	12.0579	15.816	3.3616
2020	0.195	12.078	15.831	3.372
LFL-Forecaster				
	Q1	Q2	Q3	Q4
2015	1.26282	12.6307	16.2937	5.07937
2016	1.09775	12.4344	16.2156	5.27505
2017	1.80724	12.5146	16.1768	5.49249
2018	1.64873	12.5088	16.139	5.42315
2019	1.68958	12.4704	16.0909	5.49409
2020	1.86951	12.4733	16.0489	5.57275
Real Observations				
	Q1	Q2	Q3	Q4
2015	1.6	12.1	18.2	5.73
2016	1.63	12.77	17.13	3.2
2017	0.47	12.97	16.37	4.67

The standard approach uses linear regression applied for each quarter separately; the resulting functions[3] support the rising trend of average temperatures for each quarter, i.e., the most recent average temperatures are well above the long-term averages and further increase should be expected (on average). The estimated functions are used to extrapolate future values of quarterly averages; of course, given that a linear function is assumed, the values must consistently increase over time, quarter by quarter (see top panel of Table 1).

In contrast, the LFL-Forecaster allows us to use all quarterly data at once, because its trend / cycle function can easily fit the seasonal effect as well. Following the learning period, the last 12 years are used as a validation period to avoid overfitting (Figure 2).

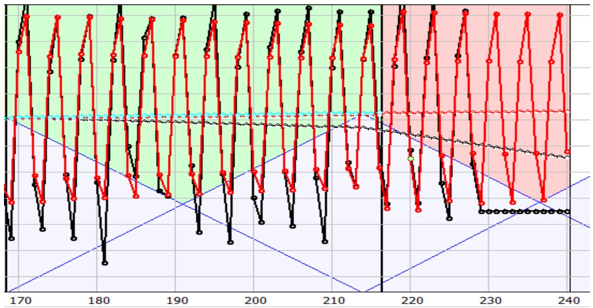

Figure 2 LFL-Forecaster cut showing validation and forecasting period (for more details, see Figure 3)

The complexity of the estimator also leads to the non-linearity of forecasted values in particular quarters (they can easily oscillate up and down, see middle panel of Table 1).

Finally, the bottom part of Table 1 shows truly observed averages in particular quarters of the first part of the forecasting period (i.e., currently known values over 2015–2017). It is apparent that the LFL-Forecaster works much better than linear regression, except for in some (mostly winter) quarters when the averages are apparently overestimated.

By applying estimated quadratic regression function (see Section 4), we can forecast quarterly sales as evident from Figure 4. These values can be further used for short- as well as long-term planning, firm value estimation and sales-at-risk calculation.

6. Conclusion

With growing pressure on performance and data regarding customer behaviour and supply chain process widely available, stock keeping units seek to optimise the level of inventories. This paper has sought to forecast sales in the Ice category when dependency on quarterly average temperatures in the form of exponential function is assumed. We particularly concentrated on LFL-Forecaster, a method combining fuzzy transform and fuzzy natural logic of fuzzy sets theory, as a tool for average temperature forecasting. The results have

[2] See the webpages of CHMU, http://portal.chmi.cz/historicka-data/pocasi/uzemni-teploty

[3] Both *t*-test as well as *F*-test show statistical significance at 5%.

been compared with simple linear extrapolation and truly observed temperatures. The utilisation of LFL-Forecaster has shown significant improvements over simplifying linear regression.

References

ARMSTRONG, J.S., GREEN, K.C., GRAEFED, A. (2015). Golden rule of forecasting: Be conservative. *Journal of Business Research* 68(8): 1717–1731. https://doi.org/10.1016/j.jbusres.2015.03.031

BOYLAN, J.E. (2010). Choosing levels of aggregation for supply chain forecasts. *Foresight* 18: 9–13.

CHEN, H., BOYLAN, J.E. (2008). Empirical evidence on individual, group and shrinkage seasonal indexes. *International Journal of Forecasting* 24: 525–534. https://doi.org/10.1016/j.ijforecast.2008.02.005

CHEN, H., BOYLAN, J.E. (2007). Use of individual and group seasonal indexes in subaggregate demand Forecasting. *Journal of the Operational Research Society* 58: 1660–1671. https://doi.org/10.1057/palgrave.jors.2602310

DAVENPORT, K. (2014). Weekly Sales Forecasting, San Diego Data Science and R Users Group, June 2014. Available from: <http://kldavenport.com/talks>

GUO, Z.X., WONG, W.K., LI, M. (2013). A multivariate intelligent decision-making model for retail sales forecasting. *Decision Support Systems* 55: 247–255. https://doi.org/10.1016/j.dss.2013.01.026

HOLČAPEK, M., TICHÝ, T. (2011). A smoothing filter based on fuzzy transform. *Fuzzy Sets and Systems* 180(1): 69–97. https://doi.org/10.1016/j.fss.2011.05.028

LEE, H.L., SO, K.C., TANG, C.S. (2000). The value of information sharing in a two-level supply chain. *Management Science* 46: 626–643. https://doi.org/10.1287/mnsc.46.5.626.12047

MA, S., FILDES, R., HUANG, T. (2016). Demand forecasting with high dimensional data: The case of SKU retail sales forecasting with intra- and inter-category promotional information. *European Journal of Operational Research* 249: 245–257. https://doi.org/10.1016/j.ejor.2015.08.029

NOVÁK, V. (2016). Linguistic characterization of time series. *Fuzzy Sets and Systems* 285: 52–72. http://dx.doi.org/10.1016/j.fss.2015.07.017

NOVÁK, V. (1995). Towards formalized integrated theory of fuzzy logic. In: Z. Bien, K. Min (Eds.): *Fuzzy Logic and Its Applications to Engineering, Information Sciences, and Intelligent Systems*, pp. 353–363. Dordrecht: Kluwer. https://doi.org/10.1007/978-94-009-0125-4_35

NOVÁK, V., PERFILIEVA, I., HOLČAPEK, M., KREINOVICH, V. (2014). Filtering out high frequencies in time series using F-transform. *Information Sciences* 274: 192–209. https://doi.org/10.1016/j.ins.2014.02.133

NOVÁK, V., ŠTĚPNIČKA, M., DVOŘÁK, A., PERFILIEVA, I., PAVLISKA, V., VAVŘÍČKOVÁ, L. (2010). Analysis of seasonal time series using fuzzy approach. *International Journal of General Systems* 39: 305–328. https://doi.org/10.1080/03081070903552965

PÁNÍK, V. (2016). *Analysis of the Influence of Selected Risks on Value of the FMCG Company*. Master Thesis, Ostrava: VSB-TU Ostrava.

PERFILIEVA, I. (2006). Fuzzy transforms: Theory and applications. *Fuzzy Sets and Systems* 157: 993–1023. https://doi.org/10.1016/j.fss.2005.11.012

PERFILIEVA, I., DAŇKOVÁ, M., BEDE, B. (2011). Towards a higher degree F-transform. *Fuzzy Sets and Systems* 180: 3–19. https://doi.org/10.1016/j.fss.2010.11.002

SYNTETOS, A.A., BABAI, Z., BOYLAN, J.E., KOLASSA, S., NIKOLOPOULOS, K. (2016). Supply chain forecasting: Theory, practice, their gap and the future. *European Journal of Operational Research* 252: 1–26. https://doi.org/10.1016/j.ejor.2015.11.010

WITHYCOMBE, R. (1989). Forecasting with combined seasonal indexes. *International Journal of Forecasting* 5: 547–552. https://doi.org/10.1016/0169-2070(89)90010-1

Appendix

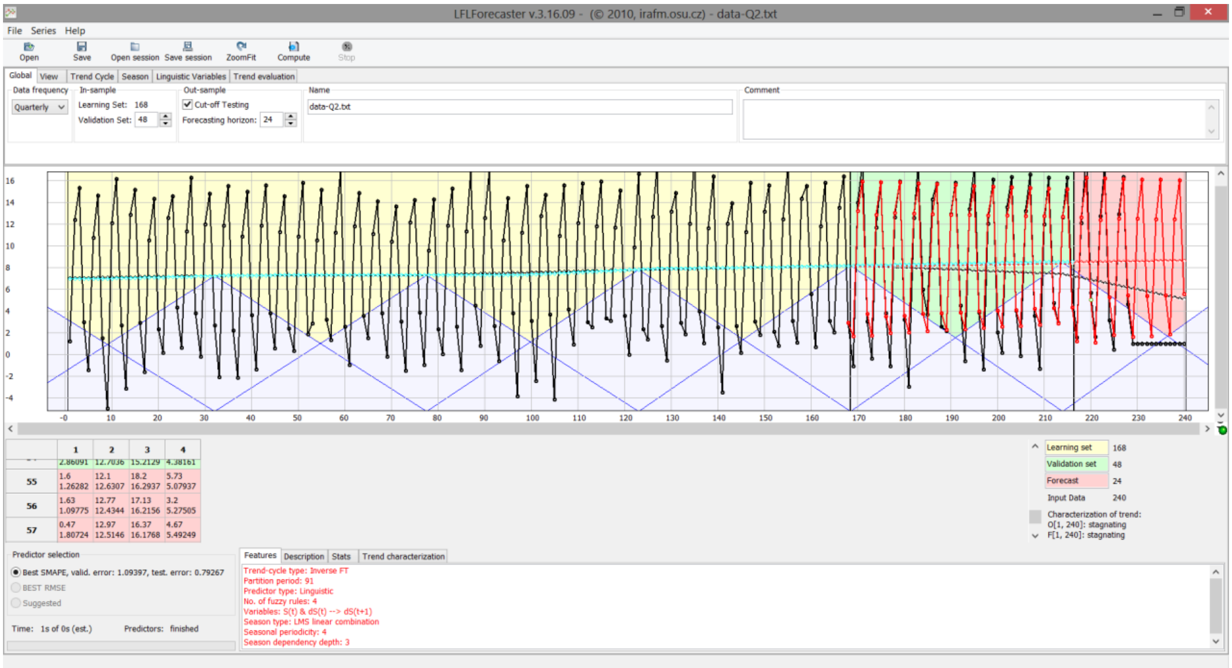

Figure 3 LFL-Forecaster showing the setting, learning set, validation set and forecasted period

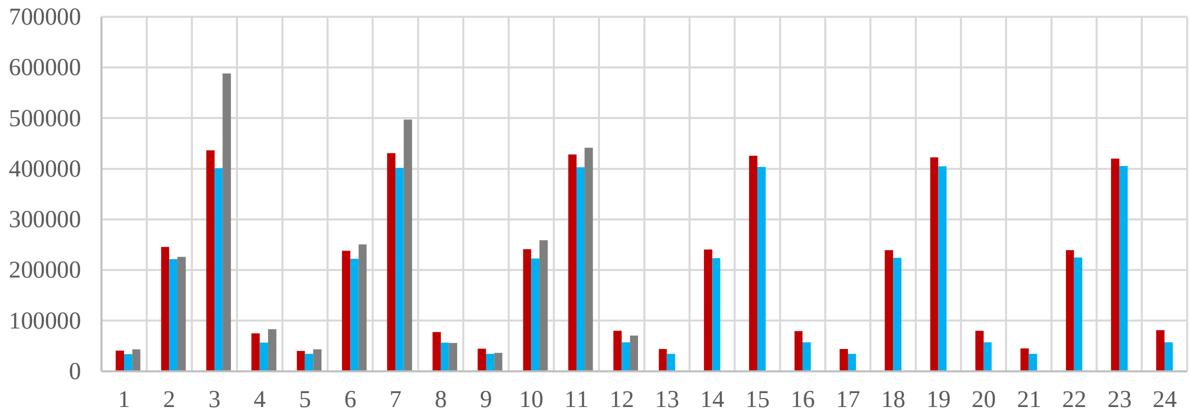

Figure 4 Quarterly sales in Ice category (2015–2020), forecasted on the basis of quarterly average temperatures obtained with LFL-Forecaster (left columns), linear regression (middle columns) and compared with truly average temperatures.