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Research on the Risk Spillover Effect Applying the EVT–Copula–CoVAR Model

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Abstract

The goal of this paper is to apply the extreme value theory, copula function and conditional value-at-risk method. Specifically, the EVT–copula–CoVaR model is constructed and combined with the Copula function to analyse the dynamic correlation between the price of gold and the world's major stock markets. On the basis of the proposed model and results, the conditional value at risk (CoVaR) and the marginal risk spillover effect (ΔCoVaR) measures are used to analyse the impact of gold prices on the world's major stock markets. The empirical results show that the fluctuation of the gold price has a certain risk spillover effect on the world's major stock markets.

Keywords

Risk spillover, volatility effect, extreme value theory, VaR, CoVaR, copula function

JEL Classification: C51, G15, G17

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1. Introduction

With the development of basic sciences, such as mathematics and statistics, and the improvement of information science and technology, risk decision making has gradually been integrated into relevant theories to help investors make analytical decisions. The most famous model among them is the value-at-risk (VaR) measurement model launched by JP Morgan, which represents the maximum possible value of a financial investment portfolio's loss in a certain period of time at a given probability level. It quantifies market risks with greater uncertainty through given standards so that the risks of different financial asset portfolios can be quantified and compared with a unified standard type.

Economic globalization inevitably increases the closeness of the economic relations between different countries and regions. With the gradual improvement of the global capital market opening level, the risks involved in the market are no longer limited to a single market. The risks of a single industry or market will most likely spread to other industries and markets through interconnected market channels, eventually leading to systemic financial risks. The contagion and spread of this risk among different industry markets is called the risk spillover effect. In the 2008 global financial crisis, which caused huge losses due to the subprime mortgage crisis, the world underestimated the scale of risk spillovers in the subprime loan and its derivative securities markets and lacked effective methods to measure extreme risks and spillover effects. The VaR model, which is commonly used by financial institutions, only captures risk probability events within a given confidence interval (usually 95% or 99%), which obviously underestimates extreme risks (that is, risk events that originate from outside the given confidence interval). The impact and analysis are limited to the risk control of a single financial institution or the financial market itself, and the impact of the risk event on the entire financial system is not considered, resulting in the quantified financial risk being far lower than the actual level. At the same time, traditional financial theories generally assume that the distribution of financial asset returns obeys a normal distribution. However, with the deepening of research in recent years, a large number of empirical data analyses has shown that the distribution of financial asset returns often has the characteristics of leptokurtosis, fat tails and skewness. The difficulties encountered in market risk control and risk quantification are mainly based on the selection of the specific distribution characteristics of the fat tail part of the return sequence, the selection of the confidence interval based on the specific distribution and the construction of extreme risk and spillover effect measurement models.

Strategic commodities and stock markets play an important role in the world's financial market system. As a "quasi-currency", gold occupies an important position in the fiscal reserves of all countries. Investors treat gold as part of their investment portfolio to diversify the risks. Therefore, in the long run, contrary to the stock market, the price of gold has an obvious negative correlation with the world's economic situation. To reduce the systemic risks caused by local risk spillovers effectively, it is necessary to consider the effective quantification of the financial risk spillover effects under extreme conditions, that is, under the conditions of an open economy, to quantify the interdependence between different types of financial markets in different countries and regions and the risk spillover effects caused by the interdependence.

A large number of studies on spillover analysis have focused primarily on the risk transmission path and the direction spillover effect without analysing the dynamics. The results of this study should help to fill this knowledge gap.

The goal of this paper is to analyse the spillover effect through the dynamic correlation between the price of gold and the world's major stock markets. To detect the intensity of the spillover effect from the dynamic of the correlation structure between given markets, the extreme value theory, copula function and conditional value-at-risk method are employed. Specifically, the EVT–copula–CoVaR model is constructed and combined with the copula function. On the basis of the proposed model and results, the conditional value at risk (CoVaR) and the marginal risk spillover effect (ΔCoVaR) measures are used to analyse the impact of gold prices on the world's major stock markets.

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2. Research Overview

The mutual influence of risks between different markets has always been a hot area for scholars to study. The gold market is known as the weather vane of the world economy due to the characteristics of gold preservation. The fluctuation of the stock market and the fluctuation of gold prices have a strong correlation. The volatility of the stock market and the gold market has brought about the transfer of wealth and the capital flow of the international capital market, resulting in the risk of different markets changing with the volatility. Research on financial market risk spillovers has certain significance for financial market supervision and the formulation of macroeconomic and monetary policies. Basher and Sadorsky (2016) pointed out in their research that gold is similar to bonds and other assets and has a better effect in hedging stock market risks. This means that there is a certain correlation between gold price fluctuations and stock market fluctuations. Similarly, Raza et al. (2016) studied the relationship between the commodity futures market and the stock market of emerging countries. They confirmed that the volatility of the commodity futures market has a negative impact on the stock market.

In view of the disadvantages of traditional value-at-risk models in measuring extreme tail risks, Adrian and Brunnermeier (2008, 2016) proposed the use of conditional value-at-risk (CoVaR) methods to quantify the risk spillovers between different institutions or markets. The CoVaR method compensates for the shortcomings of tools such as the indirect measurement of risk using only variance. Especially for the practical application of risk management, its ideas and methods are qualitative changes. Girardi and Ergun (2013) conducted a more in-depth study on the definition of CoVaR. Focusing on the more serious crisis events in the tail distribution, the multidimensional GARCH model was used to simulate the widespread risk spillover effects among financial institutions. Subsequently, many scholars used the CoVaR method proposed by Adrian and Brunnermeier (2016) to study risk spillover effects. Bernal et al. (2014) introduced ΔCoVaR to measure systemic risk and assess the degree of influence on systemic risk when problems occur in different financial sectors, such as banking and insurance. The empirical results show that, between 2004 and 2012, the financial sector, such as banks and insurance companies, in the eurozone had a relatively greater impact on systemic risk than other financial service sectors, and the banking industry in particular had a greater impact on systemic risk than the insurance industry. Castro and Ferrari (2014) used a sample of 26 large European banks to analyse empirically the contribution of CoVaR to measuring financial institutions' systemic risks. The copula function theory was first proposed by Sklar (1959). Its essence is to describe the marginal distribution of the joint distribution of random variables and the correlation structure between variables. Embrechts et al. (1999) first applied the copula function to analyse the financial market risk. They believed that the dependency index derived from the copula function was consistent with the actual situation of the financial market. In recent years, its application in the financial industry and the insurance industry has gradually expanded. The application of the extreme value theory (EVT) used to be based mainly on the climate and hydrology. In recent years, its application in the financial industry and insurance industry has gradually expanded. For the extreme value theory, the core problem is modelling extreme events scientifically and reasonably. The current literature does not contain separate research on the extreme value theory and financial market risk but uses the extreme value theory to cooperate with more different risk models to achieve the purpose of comprehensive research.

The current research on financial market risk is mostly based on the combination of the quantile regression method and the GARCH family of models to measure the VaR value, and there is little literature on combining the copula function and extreme value theory to calculate VaR. Most of the CoVaR literature has only focused on the risk transmission path and direction and has rarely involved the intensity of risk spillover, which is not conducive to a comprehensive and in-depth understanding of risks and affects risk supervision. Based on the advantages and disadvantages of the above models, this article will use the extreme value theory combined with the copula function to construct a spillover effect for quantifying risk to study the contagion of the gold market risk to the stock market risk.

3. Model Introduction

3.1 Conditional Value at Risk

Adrian and Brunnermeier (2008) were the first to propose the conditional value-at-risk (CoVaR) method. This method is a risk measurement method based on VaR to measure the risk between financial institutions, which helps to quantify the systemic risk of financial institutions and the risks of other financial institutions. The most significant difference between the VaR method and the CoVaR method is that the CoVaR value can measure the risk spillover effect of one financial institution on another financial institution. Recalling the definition of VaR provided above, if a given financial institution i has a rate of return r_t^i and a confidence level p, then VaR_{1-p}^i can be expressed as:

$$Pr\left(r_t^i \leq VaR_{1-p}^i\right) = 1 - p \tag{1}$$

(VaR_{1-p}^{i} is usually a negative value, but, in actual application, it is generally expressed as a positive value). VaR is a risk assessment of a single financial asset and cannot reflect the degree of risk spillover between financial markets or assets. Adrian and Brunnermeier (2008) proposed the concept of CoVaR on the basis of VaR. It represents the value of risk faced by financial asset i when financial asset j is at a risk level. Therefore, $CoVaR_{1-p}^{i/j}$ reflects the conditional risk of financial asset i to financial asset j, which can be expressed as

$$Pr(r_t^i \leq CoVaR_{1-p}^{i/j} | r_t^i = VaR_{1-p}^{j}) = 1 - p \quad (2)$$

It can be seen from the above formula that the essence of $CoVaR_{1-p}^{i/j}$ is a conditional VaR, which measures the total risk of financial asset i, including the risk value of financial asset i itself and the risk spillover effect of financial asset j. CoVaR reflects conditional risk and infectious spillover risk (see Brayek et al., 2015). It is mainly aimed at measuring the risk of extreme events under extreme tail probabilities. It is a conditional concept that can be used to capture the effect of risk spillovers. To evaluate the risk spillover effect of financial asset j on financial asset i, this study defines $\Delta CoVaR_{1-p}^{i/j}$. The specific formula is as follows:

$$\Delta CoVaR_{1-p}^{i/j} = CoVaR_{1-p}^{i/j} - VaR_{1-p}^{i} \quad (3)$$

Considering that the VaR of different financial assets has relatively large differences and $\Delta CoVaR_{1-p}^{i/j}$ can only indicate the size of the risk spillover effect, it is necessary to standardize $\Delta CoVaR_{1-p}^{i/j}$ to reflect the strength of the spillover effect of financial assets:

$$\%CoVaR_{1-p}^{i/j} = \frac{\Delta CoVaR_{1-p}^{i/j}}{VaR_{1-p}^{i}} \times 100\% \quad (4)$$

3.2 Extreme Value Theory

The extreme value theory deals with extreme situations of risk. It has the ability to estimate beyond sample data and can accurately describe the tail distribution. In a statistical sense, extreme values refer to maximum and minimum values. Although the extreme values in some data sets do not have a large gap from other data, there are still extreme values in this data set.

The extreme value theory mainly consists of two types of models, namely the traditional block maxima method (BMM) model and the peak-over-threshold (POT) model. The difference between these two types of extreme values lies in the selection of data. The POT model has a predetermined threshold. When there are data that exceed the threshold, they will be acquired and formed into a new group, using a new sequence to model. The BMM model is different: the initial data will be classified, and then the maximum value in each group will be obtained to form a new group, and the new group will be used for modelling. However, both models work with extreme data in the tail rather than analysing the overall distribution.

The BMM model involves maximum likelihood estimation and the probability weighted moment estimation method. In its use, a large amount of sample data is often needed to model the maximum value after the block. Due to the limited acquisition of tail data, this method has great application difficulties in practice. Therefore, the peak-over-threshold (POT) model is used more in practice.

POT is a key branch of the extreme value theory. Its main feature is the ability to model all the observations in a sample that exceed a sufficiently large threshold. The form is simple, easy to calculate and has a wide range of applications. The POT model has two types of methods: one is the semiparametric method based on a Hill-type estimator; the other is the full parameter method based on the generalized Pareto distribution.

The semiparametric method is used when the shape parameter of the tail distribution $\xi > 0$, and the Hill-type estimator is used to estimate the tail index $\alpha = 1/\xi$. If $\xi > 0$, $L(x)$ is a slow-changing function; if and only if $\bar{F}(x) = 1 - F(x) = x^{1/\xi} L(x)$, then $F \in MDA(H)$. Suppose that $X_1, X_2, \ldots, X_{n\_}$, n is a sample from the population distribution $F(X)$ and its order statistic is $X_{(n)} \geq \ldots \geq X_{(n-k)} \geq \ldots \geq X_{(2)} \geq X_{(1)}$, where $X_{(n-k)}$ is a larger observation value and there are k sample points greater than $X_{(n-k)}$. Hill (1975) gave an estimate of α:

$$\hat{\alpha} = [\frac{1}{k} \sum_{i=n-k+1}^{n} (lnX_i - lnX_{n-k})]^{-1}, 2 \leq k \leq n \quad (5)$$

It can be seen from the above formula that $\hat{\alpha}$ depends on the sample points greater than a certain threshold $X_{(n-k)}$. Therefore, the choice of $X_{(n-k)}$ is the key to estimating $\hat{\alpha}$ correctly. There are methods such as the excess expectation function graph, Hill graph, Du Mouchel 10% principle and so on. Taking the Hill chart as an example, in practical applications, it is used to determine k, that is, $X_{(n-k)}$. Taking $k, (k = 2, \ldots, n)$ as the abscissa and $\hat{\alpha}$ as the ordinate to draw the plot, and selecting the data corresponding to coordinate k of the starting point of the stable region of the tail index in the Hill graph, $X_{(n-k)}$ serves as the threshold u.

Different from the semiparametric method, the full-parametric method uses the generalized Pareto distribution to simulate the tail distribution that exceeds the threshold and then estimate its shape parameter ξ. It is assumed that the excess values are mutually independent and obey the generalized Pareto distribution; the time when the excess value occurs obeys the Poisson distribution; at the same time, the excess value and the generation time of the excess value are independent of each other. Assuming that $F(X)$ is the distribution function of financial asset loss and that u is a sufficiently large threshold, then $Y = X - \mu$ is called the excess loss, and its distribution function can be recorded as:

$$F_u(y) = P(X - u \leq |X > u),\ \ 0 \leq y \leq X_F - u \qquad (6)$$

Among them, $X_F = sum\{X_F \in R: F(X) < 1\} \leq \infty$ is the right endpoint of $F(X)$. The out-of-limit distribution function represents the probability that the loss exceeds the threshold. The larger y value gives the loss that exceeds the threshold. The multiplication formula is defined as follows (Wang et al., 2018):

$$\mathrm{F_u(y)} = \frac{\mathrm{F(y+u)-F(U)}}{\mathrm{1-F(u)}} \qquad (7)$$

By simplifying the above formula, we can obtain the final distribution function of financial asset loss:

$$F(x) = F(y+u) = F_u(y)\big(1 - F(u)\big) + F(u),$$
$$x > u \qquad (8)$$

According to the previous derivation and the Fisher–Tippett theorem, it can be shown that, if the distribution of the maximum value sequence is known to converge, its limit distribution can be transformed into a generalized extreme value distribution $(H_{\xi,\mu,\sigma(x)})$ with a specific value of the parameter α, μ, δ. In addition, according to the results of Balkem and Haan (1974) and Pickands (1975), if F belongs to the maximum attractive field of H, then the generalized Pareto distribution is the limit distribution of the over-limit distribution, that is, $X \sim F, \xi \in R, F \in MDA(H)$; if and only if there is a certain positive measure function $\beta(u)$, the following limit theorem applies:

$$\lim_{u \to X_F} \mathop{\mathrm{SUP}}_{0 \leq Y \leq X_F - \mu} \left| F_u(y) - G_{\xi,\beta(u)}(y) \right| = 0 \qquad (9)$$

where $G_{\xi,\beta(u)}(y)$ is a generalized Pareto distribution. The above formula shows that, for a sufficiently large threshold, the overrun distribution function can be approximated by the generalized Pareto distribution. The generalized Pareto distribution is defined as follows:

$$F_u(y) \approx G_{\xi,\beta}(y) = \begin{cases} 1 - (1 + \xi\frac{y}{\beta})^{-1/\xi}, \xi \neq 0 \\ 1 - e^{-y/\beta},\ \xi = 0 \end{cases} \qquad (10)$$

where ξ is the shape parameter, β is the scale parameter and $\beta > 0$; when $\xi > 0$, $x \geq 0$; and, when $\xi < 0$, $0 \leq x \leq -\beta/\xi$. When $\xi > 0$, the generalized Pareto distribution corresponds to the thick-tailed ordinary Pareto distribution, which is the most relevant to risk measurement; when $\xi = 0$, it corresponds to an exponential distribution; and, when $\xi < 0$, it corresponds to a short-tailed distribution, such as a uniform distribution. The parameters ξ and β are unknown and need to be estimated based on excess loss data.

There are many methods to estimate ξ and β, such as maximum likelihood estimation, the moment estimation method and so on. The maximum likelihood estimation method is the most commonly used method. Supposing that it is taken from the sample data $X_1, X_2, \ldots, X_n$, the population distribution is F. The sample points larger than the threshold are recorded as $\hat{X}_1, \hat{X}_2, \ldots, \hat{X}_n$, and there are N_u sample points in total. The over-limit value $y_i = \hat{X}_j - u$ is calculated. From the above definition of the generalized Pareto distribution, the density function of the generalized Pareto distribution can be obtained:

$$g'_{\xi,\sigma}(y) = \frac{1}{\beta}(1 + \frac{\xi}{\beta}y)^{-(1+1/\xi)} \qquad (11)$$

Among them, when $\xi > 0$, $y \geq 0$; when $\xi < 0$, $0 \leq x \leq -\beta/\xi$, and its log likelihood function is

$$l(\xi, \beta; y) = -N_u ln\beta - \left(1 + \frac{1}{\beta}\right)\sum_{i=1}^{N_u} ln(1 + \frac{\xi}{\beta}y_i) \qquad (12)$$

Under the premise of the likelihood function, the likelihood equation can be derived:

$$\begin{cases} \frac{\partial l}{\partial \beta} = -\frac{N_u}{\beta} + (1+\beta)\sum_{i=1}^{N_u}\frac{y_i}{\beta(\beta+\xi y_i)} \\ \frac{\partial l}{\partial \xi} = \frac{1}{\xi^2}\sum_{i=1}^{N_u} ln(1 + \frac{\xi}{\beta}y_i) - (1+\beta)\sum_{i=1}^{N_u}\frac{y_i}{\beta(\beta+\xi y_i)} \end{cases} \qquad (13)$$

Let the above two equations be equal to zero; then, the maximum likelihood estimates of parameters ξ and β can be obtained.

3.3 Copula function definition and related theorems

A copula function is a kind of function cluster that connects joint distribution functions with their respective marginal distribution functions. It was first proposed by Sklar (1959). With the development of modern information technology, it began to be applied to the financial field in the late 1990s.

Definition: The copula function is a connection function that connects the joint distribution function of d random vectors with their respective edge distribution functions. If a function C satisfies (McNeil et al., 2005)

1. $C: [0,1]^d \to [0,1]$,
2. $C(u_1, u_2, \ldots, u_d)$ increases monotonically with respect to u_i, $i \in \{1,2,..,d\}$,
3. To all $u_i \in [0,1], i \in \{1,2,\ldots,d\}$, $C(1,\ldots,1,u_i,1,\ldots,1) = u_i$ exists;
4. To all $(a_1,\ldots,a_d),(b_1,\ldots,b_d) \in [0,1]^d$ and $a_i \leq b_i$, $\sum_{i_1=1}^{2} \cdots \sum_{i_2=1}^{2}(-1)^{\cdots\ldots+i_d} C(u_{1\ldots},\ldots,u_{di_d}) \geq 0$ exists, where $u_{j1} = a_j, u_{j2} = b_j, j \in \{1,2,\ldots,d\}$.

Theorem (Sklar, 1959): If there are d random variables, $F(x_1,\ldots,x_d)$ is the $d - ary$ joint distribution function with marginal distribution function $F_1(x_1),\ldots,F_d(x_d)$; then, there is a copula function

$C(u_1, u_2, \ldots, u_d)$ that makes the following equation true:

$$F(x_1, \ldots, x_d) = C(F_1(x_1), \ldots, F_d(x_d)) \quad (14)$$

If $F_1(x_1), \ldots, F_d(x_d)$ is a continuous function, then $C(u_1, u_2, \ldots, u_d)$ is uniquely determined; on the contrary, if $F_1(x_1), \ldots, F_d(x_d)$ is a univariate distribution function, then $C(u_1, u_2, \ldots, u_d)$ is a copula function. On this basis, $F(x_1, \ldots, x_d)$ determined by $F(x_1, \ldots, x_d) = C(F_1(x_1), \ldots, F_d(x_d))$ is a $d-ary$ joint distribution function with edge distribution $F_1(x_1), \ldots, F_d(x_d)$.

1. In addition, the copula function has the following properties:
2. $C(u_1, u_2, \ldots, u_d)$ is monotonous and non-decreasing with respect to each variable,
3. $C(u_1, u_2, \ldots, 0, \ldots, u_d) = 0, C(1, \ldots, 1, u_i, 1, \ldots, 1) = u_i$,
4. Any $u_i, v_i \in [0,1](i = 1,2, \ldots, d)$ has $|C(u_1, u_2, \ldots, u_d) - C(v_1, v_2, \ldots, v_d)| \leq \sum_{i=1}^{d} |u_i - v_i|$
5. Make $C^-(u_1, u_2, \ldots, u_d) = max\ (\sum_{i=1}^{d} u_i - N + 1, 0)$, $C^+(u_1, u_2, \ldots, u_d) = min\ (u_1, u_2, \ldots, u_d)$; then, for any $u_i \in [0,1](i = 1,2, \ldots, d)$, there is: $C^-(u_1, u_2, \ldots, u_d) \leq C(u_1, u_2, \ldots, u_d) \leq C^+(u_1, u_2, \ldots, u_d)$, denoted as $C^- \prec C \prec C^+$. Call C^- and C^+ the lower and upper bounds of Frechet, respectively, where $d \geq 2, C^+$ is a $d-ary$ copula function, but, when $d > 2$, C^- is not a copula function.
6. If $U_i \sim U(0,1), (i = 1,2, \ldots, d)$ are independent of each other; then, $C(u_1, u_2, \ldots, u_d) = \prod_{i=1}^{d} u_i$.

Before actually using the copula theory, it is necessary to estimate the unknown parameters. Assuming that the joint distribution function of the random vector (X, Y) is $F(X, Y)$, the joint density function is $f(x, y)$, the marginal distribution functions are $F_1(X)$ and $F_2(Y)$, respectively, the marginal density functions are $f_1(x)$ and $f_2(y)$ and the density function of the corresponding copula function $C(u, v)$ is $c(u, v)$. Then,

$$f(x, y) = c(F_1(X), F_2(Y)) f_1(x) f_2(y) \quad (15)$$

where $c(u, v) = \frac{\partial^2 C(u,v)}{\partial u \partial v}$. The above formula shows that a joint density function $f(x, y)$ can be decomposed into two parts: one part $c(u, v)$ becomes the density function of the corresponding copula function C(u,v), which reflects the dependence structure of random variables X and Y; the other part is the product $f_1(x) f_2(y)$ of the edge density function. Related theories were proposed by Sahamkhadam et al. (2018). Assuming that $\{(X_i, Y_i), i = 1,2, \ldots, n\}$ is a sample of a random vector (X, Y), then its log-likelihood function is the following:

$$l = \sum_{i=1}^{n} lnc(F_1(X_i), F_2(Y_i)) + \sum_{i=1}^{n} [lnf_1(X_i) + lnf_2(Y_i)] \quad (16)$$

It can be seen from the above formula that, as long as the edge distribution function and the edge density function of the random vector (X, Y) are known, the relevant parameters can be obtained on this basis.

After clarifying the marginal distribution and selecting the optimal copula function, the spillover effect of financial assets is calculated according to the definition of CoVaR. The specific process can be described in the following way. First, suppose that there is $(U, V) \sim C$, where C is the copula function, $F_{i,j}(x^i, x^j)$ represents the distribution function of the joint distribution, U indicates the marginal distribution function $F_i(x^i)$ of financial assets X^i and V represents the marginal distribution function $F_j(x^j)$ of financial assets X^j. The corresponding density functions are $f_{i,j}(x^i, x^j)$, $f_i(x^i)$ and $f_j(x^j)$. Thus, the form of the conditional distribution density function of financial assets X^i is as follows:

$$f_{i|j}(x^i | x^j) = \frac{f_{i,j}(x^i, x^j)}{f_j(x^j)} \quad (17)$$

According to Sklar's theorem, the above formula can be derived as below:

$$f_{i|j}(x^i | x^j) = c(f_i(x^i), f_j(x^j)) f_i(x^i) \quad (18)$$

The distribution function of the above formula is

$$F_{i|j}(x^i | x^j) = \int_{-\infty}^{x^i} c(F_i(x^i), F_j(x^j)) f_i(x^i) dx^i \quad (19)$$

The marginal distribution functions $F_i(x^i)$ and $F_j(x^j)$ in the above formula can be obtained using the previous beta-skew-t-EGARCH-EVT model, the density function $f_i(x^i)$ is the derivative of the marginal distribution function $F_i(x^i)$ and c is the density function of the optimal copula function selected above. According to Mainik (2012), $CoVaR_{1-p}^{i/j}$ can be expressed as

$$CoVaR_{1-p}^{i/j} = F_{i|j}^{-1}(1 - p | VaR_{1-p}^{j}) \quad (20)$$

In the above formula, $F_{i|j}^{-1}$ is the inverse function of $F_{i|j}$. Under normal circumstances, its analytical solution is difficult to find, so, to solve it, it is converted into the following expression:

$$\int_{-\infty}^{x^i} c(F_i(x^i), F_j(x^j)) f_i(x^i) dx^i = 1 - p \quad (21)$$

That is, solution x^i of the above formula is $CoVaR_{1-p}^{i/j}$.

4. Empirical Analysis

To study the dynamic correlation and risk spillover effects between the gold market and the stock markets of different countries and regions, this paper selects the daily data of the gold spot price rate of return and the rate of return of major international stock market indexes (Standard & Poor's Index (S&P500), Nasdaq Index (Nasdaq), Dow Jones Index (Dow Jones), London FTSE100 Index (FTSE), Paris CAC Index (CAC40), Germany DAX40 Index (DAX40), Nikkei 225 Index (N225), Hong Kong Hang Seng Index (HSI), Shanghai Composite Index (SHZ) and Shenzhen Component Index (SHE)) as the research samples, the data source being Yahoo Finance. Considering that a longer data interval can better reflect the spillover effects between different markets, the collection period is from 15 November 2005 to 15 December 2019. The paper takes the first-order logarithmic difference of the gold spot price and the stock market index during the sampling period to calculate the daily return rate and multiplies the return rate result by 100 to reduce the error:

$$R_t = \ln\,(p_t/p_{t-1}) \times 100$$

After eliminating invalid data, the descriptive statistical results of the data shown in the following table are calculated: the skewness coefficient of the gold price return sequence and the return sequence of each stock market index are close to the skewness coefficient 0 corresponding to the normal distribution, and the kurtosis coefficient is much larger than 3, corresponding to the normal distribution. At the same time, the Jarque–Bera test result of the return rate sequence shows that the probability value p is 0; that is, the gold price return rate sequence and the stock market index return rate sequence are significantly different from the normal distribution at the 5% significance level. Therefore, it can be judged preliminarily that neither the gold price return rate sequence nor the stock market index return rate sequence obeys the normal distribution, see Appendix (Table 1).

In the following research, to conduct a deeper study on the return rate of the gold price and the return rate sequence of each stock market index, a Q-Q plot corresponding to each return rate sequence is constructed. Due to space limitations, only the Q-Q plot of the HSI data is shown here. It is clear from the Figure 1 that the upper and lower tails of the HIS yield deviate significantly from the normal distribution and have significant fat tail characteristics. The Q-Q plot test on other return sequence data selected in this article obtains similar conclusions. Combining the kurtosis values of each return sequence in the above table, it can be concluded that, for the selected data, the gold price return rate and the return rate data of each stock market index generally have significant kurtosis and fat tail characteristics.

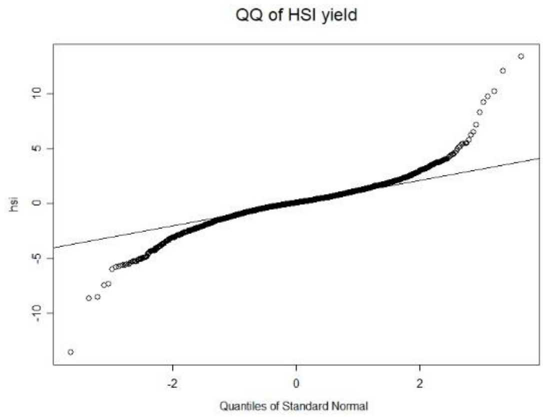

Figure 1 QQ plot of the HSI sequence

According to the relevant information of the extreme value theory introduced above, the generalized Pareto distribution within the frame of extreme value theory can better fit the tail distribution of the return sequence. This article uses Du Mouchel's 10% threshold selection criterion to determine the upper- and lower-tail thresholds of each rate of return. After obtaining the upper- and lower-tail thresholds, the generalized Pareto distribution is used to fit the selected upper and lower tails, and the empirical distribution is used to fit the intermediate data between the upper and the lower tails. The scale parameter $\beta(u)$ and shape parameter ξ of the corresponding generalized Pareto distribution are estimated using the maximum likelihood estimation method. Still taking the HSI data as an example, the figure below shows the GPD distribution fitting diagnosis chart based on the HSI data. As shown in the figure, most of the points are concentrated near the distribution curve (including the over-threshold distribution curve and the tail distribution curve). Only a few points deviate, and these do not affect the fitting effect. The model calculation results show that the model fits the data well. The same results can be obtained by performing the same fitting on other data series and testing their effects. For space reasons, they are not repeated here.

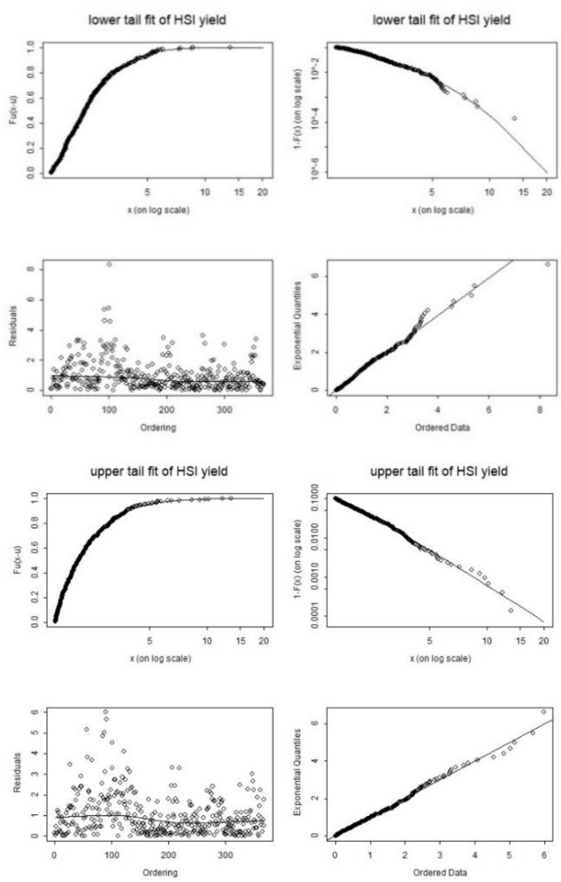

Figure 2 GPD distribution fitting diagnosis of the return rate of the HSI series (upper and lower tails)

This paper uses the extreme value theory to model the gold price return rate and the main stock market index return rate. After determining its marginal distribution, the copula function is used to characterize the dependent structure relationship of the sample data sequence. Substituting the estimated value of the parameter into the above-mentioned conditional distribution density function formula can obtain the marginal distribution function of the return rate of each stock market index. After determining the marginal distribution of the stock index return rate of the selected stock market, the copula function can be used to capture the correlation structure of each stock index return rate sequence and the gold price return rate sequence. Using the principle of the maximum likelihood function value loglike and minimum AIC, BIC and HB, the best-fitting function from the commonly used copula functions is selected and the copula function fitting results of the gold price return rate and HIS index return rate are taken as an example:

```
> compare.copulaFit(cop.gumbel.fit, cop.joe.fit,cop.frank.fit,cop.kimeldorf.sampson.fit,cop.tawn
                            loglike        AIC        BIC         HQ
            cop.gumbel.fit  9.677186 -17.354373 -11.139096 -15.142382
               cop.joe.fit  6.949974 -11.899947  -5.684670  -9.687956
             cop.frank.fit  5.415838  -8.831676  -2.616399  -6.619685
cop.kimeldorf.sampson.fit 13.195655 -24.391310 -18.176034 -22.179320
              cop.tawn.fit 17.171355 -28.342710  -9.696879 -21.706737
               cop.bb1.fit 15.478975 -26.957951 -14.527397 -22.533969
               cop.bb3.fit 49.406472 -94.812944 -82.382390 -90.388962
               cop.bb4.fit 13.230301 -22.460602 -10.030048 -18.036620
               cop.bb5.fit  9.677186 -15.354373  -2.923819 -10.930391
               cop.bb6.fit  6.949974  -9.899947   2.530607  -5.475965
               cop.bb7.fit 16.055912 -28.111824 -15.681270 -23.687842
            cop.normal.fit  5.204585  -8.409169  -2.193892  -6.197178
               cop.bb2.fit 13.237495 -22.474989 -10.044435 -18.051007
> |
```

Figure 3 Fit results of each copula function

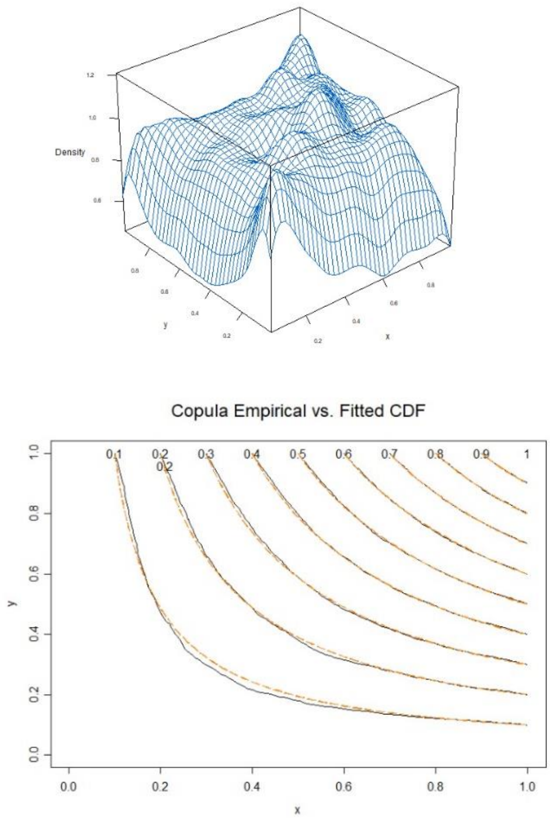

Figure 4 Density and fit result of the BB3 copula

According to the above results, the optimal copula function is the BB3 copula and, on this basis, the estimated parameters are substituted into the BB3 copula function; the BB3 copula distribution function is obtained as follows:

$$C(u,v) = exp\{-[\delta^{-1}ln(e^{\delta(-ln(u)^{\theta}} + e^{\delta(-ln(v)^{\theta}} - 1]^{\frac{1}{\theta}}\}, \theta > 1, \delta > 0$$

From the distribution function of the BB3 copula function, the density function can be obtained, so the density function can be estimated with maximum likelihood and the Kendall τ correlation coefficient and the upper- and lower-tail correlation coefficients of the gold price return rate and the return rate of each stock market index can be calculated. The results are shown in Appendix (Table 2).

The correlation between the gold market and each stock market is characterized by the BB3 copula function, and the Kendall τ correlation coefficients are all positive, indicating that the markets have strong positive dependence and the risk of risk spillover may be high. The lower-tail coefficients are all greater than zero, indicating that the risk spillover effects between the data samples are all positive and different. This result may be related to the size of the stock market and the maturity of the local financial market and market preferences. So far, this article has established the copula dependency structure function of the gold price return rate sequence and other stock market index return rates. To compare the strength of the spillover effect of the gold market risk on the stock market, we use the method mentioned above to calculate the CoVaR, ΔCoVaR and %CoVaR of each stock index return sequence at the 5% significant level when the gold market is at risk. The results are shown in Appendix (Table 3).

It can be seen from the results in the above table that, at a given confidence interval, the SZSE has the largest value at risk and the Dow Jones has the smallest value at risk. The risk price of the Asian stock market is significantly higher than that of stock markets in Europe and the United States. From the perspective of risk spillover effects, the conditional value at risk is greater than the value at risk, and there is an obvious positive spillover effect. From the perspective of risk spillover intensity, which is %CoVaR, the highest spillover intensity is the 8.7974% of the SZSE, and the lowest is the 0.1139% of the DAX. From the perspective of market division, the market spillover intensity of the Shanghai Stock Exchange Index, the Shenzhen Stock Exchange Index, the Hong Kong Stock Exchange and the stock exchanges in the United States and the United Kingdom is significantly greater than that of the French, German and Japanese markets. This may be because the stock markets of China and the United States are larger than the other markets and have greater capital liquidity. It can be seen that actual risk management usually only considers the value at risk, with which it is easy to underestimate the risks faced by the market. Taking the risk overflow of gold to the SZSE as an example, the actual value at risk, that is, the conditional value at risk, is significantly greater than the theoretical value at risk. This is only a risk overflow caused by a change in the gold market. It can be seen that the traditional value-at-risk model seriously underestimates the risk, which is likely to cause great uncertainty in risk management.

Finally, this paper draws on the model validity test method, which has the following specific form. According to the marginal distribution model and the optimal copula function calculated above, 50,000 sets of (X^i, X^j) values are randomly generated. Then, the 200 values closest to VaR_q^j are randomly selected from the randomly generated X^j values, and, at the same time, the 200 X^i values corresponding to X^j are obtained, marked as M, so that the ratio of the number of data fewer than $CoVaR_{1-p}^{i|j}$ in M to the total number of data in X is the obtained test value. The test values of the article's data results are shown in the Appendix (Table 4).

The posterior test results show that the EVT–copula–CoVaR model fits the correlation structure between the gold market and the stock market better. Financial institutions and regulatory authorities can use this model method to evaluate the direction and intensity of risk spillovers effectively when risk events occur in other financial institutions (or financial markets) and further improve their risk management decision-making capabilities.

5. Conclusion

Through the above research, it is found that the EVT–copula model can effectively fit the relevant structure of financial markets under extreme market conditions. This article combines the analysis characteristics of these two models to construct the EVT–copula–CoVaR model. The generalized Pareto distribution is used to fit the upper and lower tails of each stock market's index return sequence, while the data in the middle of the upper and lower tails of the stock index return sequence are fitted with an empirical distribution. Employing the Kendall τ correlation coefficient and the upper- and lower-tail correlation coefficients (mainly focusing on the lower-tail correlation coefficient), qualitative analysis is conducted on the risk spillover effects of gold price fluctuations on major stock indexes. The analysis based on this model shows that the gold market has a certain risk spillover effect on the world's major stock markets. Model diagnosis and posterior testing show that the model's method can effectively measure the risk spillover of a single financial institution (or financial market), which is beneficial to financial institutions, investors and financial regulatory authorities, enabling them to track changes in systemic risks in a timely manner.

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Appendix

Table 1 Descriptive statistics of the data

	Mean	*Max*	*Min*	*StD*	*SKEW*	*KURT*	*J-B test(P-value)*
Gold	0.0350	11.9691	-11.7970	1.5315	-0.0818	12.6194	0
HSI	0.0158	13.4068	-13.5820	1.4841	-0.0344	8.9301	0
SZSE	0.0451	9.1615	-9.7500	1.8630	-0.5630	3.0551	0
SSE	0.0308	9.0343	-9.2562	1.6145	-0.6448	4.6782	0
Nikkei225	0.0171	13.2346	-12.1110	1.5040	-0.4525	7.7674	0
Nasdaq	0.0458	11.1594	-13.1492	1.3783	-0.4945	9.3497	0
CAC40	0.0051	10.5946	-13.0983	1.4180	-0.2710	8.1759	0
DAX40	0.0248	10.7975	-13.0549	1.3894	-0.2373	8.2059	0
DowJones	0.0271	10.7643	-13.8418	1.2239	-0.4947	16.1341	0
S&P500	0.0287	10.9572	-12.7652	1.2766	-0.5613	13.6087	0
FTSE100	0.0048	9.3842	-11.5124	1.2016	-0.4094	9.8378	0

Source: own calculations

Table 2 Fit results of the stock index return rate and gold's dependence structure

Data sequence	*Optimal Copula*	θ	δ	*Kendall* τ	*Lower tail coefficient*	*Upper tail coefficient*
HSI	BB3 Copula	1.0763	0.0932	0.1130	0.0956	0.0960
SZSE	BB3 Copula	1.0651	0.0669	0.0920	0.1957	0.0830
SSE	BB3 Copula	1.0714	0.0710	0.0991	0.0912	0.0903
Nikkei225	BB3 Copula	1.0559	0.0494	0.0761	0.0724	0.0721
Nasdaq	BB3 Copula	1.0707	0.0606	0.0939	0.0876	0.0895
CAC40	BB3 Copula	1.0626	0.0689	0.0907	0.0847	0.0800
DAX40	BB3 Copula	1.0677	0.0727	0.0967	0.0930	0.0860
DowJones	BB3 Copula	1.0670	0.0573	0.0892	0.0791	0.0851
S&P500	BB3 Copula	1.0753	0.0682	0.1012	0.1179	0.0948
FTSE100	BB3 Copula	1.0850	0.1082	0.1265	0.0970	0.1057

Source: own calculations

Table 3 Results of VaR, CoVaR, ΔCoVaR and %CoVaR

Data sequence	*VaR*	*CoVaR*	*ΔCoVaR*	*%CoVaR*
HSI	-2.3341	-2.3804	0.0462	1.9807%
SZSE	-3.0678	-3.3377	0.2699	8.7974%
SSE	-2.6463	-2.7008	0.0545	2.0590%
Nikkei225	-2.3444	-2.3653	0.0210	0.8939%
Nasdaq	-2.1645	-2.2117	0.0472	2.1800%
CAC40	-2.2170	-2.2444	0.0274	1.2368%
DAX40	-2.2170	-2.2145	0.0025	0.1139%
DowJones	-1.8191	-1.8593	0.0402	2.2103%
S&P500	-1.9182	-1.8593	0.0589	3.0719%
FTSE100	-1.8422	-1.8939	0.0517	2.8073%

Table 4 Result of validity test

HSI	*SZSE*	*SSE*	*Nikkei225*	*Nasdaq*	*CAC40*	*DAX40*	*DowJones*	*S&P500*	*FTSE100*
5.60%	4.50%	4.80%	5.50%	4.90%	5.10%	5.50%	4.70%	4.80%	5.10%

Source: own calculations