

# Immigration to the EU and challenges for demographic modelling 

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#### Abstract

The aim of the paper is twofold: first to discuss the current immigration wave to the European Union and second to foresee the impacts of this wave on demographic modelling tools. Demographic modelling has been developing from the component method through expert estimations and expectations to become the most sophisticated LeeCarter models so far based on principal component analysis and stochastic modelling, modified gravity models or human capital models. However, unlike expert estimations, all the models are based on historical data and thus are not able to take into account unusual situations such as the recent immigration crisis. The paper describes the immigration to four terminal countries for immigrants in the EU - France, Belgium, Germany and Italy - and applies the Lee-Carter method to project the migration process in those countries. Due to a weak database, the projection of the immigration process is made only for the years 2015 and 2016 (at present still unknown). The results show a multiple increase in the immigration profiles, which will affect the current age- and sex-specific structures of the analysed populations.


## Keywords

Demographic projection, European Union, immigration, Lee-Carter method.
JEL Classification: C32, J11, J61

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# Immigration to the EU and challenges for demographic modelling 

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## 1. Introduction

It is a well-known fact that the European population is ageing rapidly. Some of the population pressures might be relieved by immigration, but it must not be too high, as, with growing disparities between the levels of material wealth in rich and poor countries, migration appears to be an attractive option for inhabitants of less developed countries (Rowlands, 1999). The increased migration from countries affected by wars and poor economic situations has recently aroused many concerns. Beside the policy consequences, the significant number of incoming people can influence not only the population structure from the nationality point of view but also the age and gender structure. It can also affect the labour market in the receiving countries. According to Moreno-Galbis and Tritah (2016), the increasing contribution of immigrants to the labour force is the most important labour supply shock that EU labour markets are facing. Therefore, some Member States have applied various restrictions. For example, Italy has set immigration quotas that can only be filled by migrants with a job offer in Italy (Campaniello, 2014).

Demographic analyses and population projections are very important. On the basis of the data on the sex and age structure of the population, it is possible to anticipate relatively well the long-term development and the future requirements, for instance in the fields of education, the health service, social services and so on (Fiala et al., 2009). Similarly, Lassilla et al. (2014) demonstrated that, although demographic forecasts are uncertain, they contain enough information to be useful in forward-looking policy rules. However, the models are typically made under the assumption that future demographic development is deterministic, and the immigration crisis can represent serious distortions to the models. In addition, the longer is the projection horizon, the less probable it is that the assumptions of the models will hold. Therefore, the stochastic approach must be considered. Despite the fact that it enables more precise projections of the population, its assumptions might not hold in the real-life turbulent development.

Migration is a very important demographic component, and it cannot be ignored in models or considered as unchangeable over time (or at least not in
the case of the larger areas). Currently the EU is facing problems of immigration. The problem of possible data set distortion should be discussed, as many demographic models are not able to explain the migration process adequately. Population migration involves the relocation of individuals, households or moving groups between geographical locations (Vitanov and Vitanov, 2016). Therefore, the aim of the paper is twofold: first to discuss the current immigration wave to the European Union and second to foresee the impacts of this wave on demographic modelling tools. The structure of the paper is as follows. Firstly, the methods and the data about migration are presented. The following section discusses the development of migration to the selected terminal states of the European Union described (France, Belgium, Germany and Italy) and the models for demographic predictions that are performed and evaluated. The last section concludes. Among the most important results of our study is the finding that the modern demographic approach that is currently used for the process of mortality and fertility is also applicable to the migration process in terminal states of the EU, with certain limitations. The best and most credible results are achieved in the case of France, because it has the highest-quality database.

### 1.1 Migration modelling

From the methodological point of view, deterministic and stochastic projections are distinguished. The first are based on predetermined assumptions. Initially they were performed by expert expectations and later by the supporting statistical methods, which were further developed and refined. Expert expectations include estimates based on the recommendations of demographers and experts from the fields of sociology, political science, medicine and law. Experts are required to provide evaluations, in the form of conditional and unconditional scenarios, of summary indicators of the demographic components determining the population evolution: that is, fertility, mortality and migration (Billari et al., 2014).

The simplest population projection can be made by log-linear regression of the population development trend. The most frequently used approach to deterministic demographic projections is the cohort
component method (e.g. Leslie, 1945 or Keyfitz, 1964), which can be enriched by the elements of the theory of probability. Cohort component models are often used to model the evolution of an age-specific population and are particularly useful for highlighting the demographic component that contributes the most to the population change (Shang et al., 2016). The algorithm of the method is old but still popular for projection thanks to its usefulness and simplicity. For example, a simple Leslie projection matrix requires only knowledge of the age structure and age-specific birth and death rates. However, the results are not robust when the population changes are large or affected by other variables.

The Box-Jenkins approach (Box and Jenkins, 1970) can also be used, as Pflaumer (1992) proved that the forecasting accuracy of the population forecasts is at least as reliable as those performed with more traditional demographic methods. The methodology projects the future values based on long time series data using seasonal autoregressive integrated moving average (SARIMA) models. Its advantage is that it is not data demanding, but sometimes it is difficult to elaborate the appropriate type of model.

Myrskylä and Godstein (2013) show that Hernes, Gompertz and logistic models, despite having been used deterministically in the past, can be improved by introducing randomness and uncertainty into the standard differential equations governing population processes. Precisely in the consideration of the randomness of processes lies the difference between deterministic and stochastic approaches.

Stochastic projections are based on stochastic modelling of time series of age-specific demographic rates and are complemented by multivariate statistical methods. The availability of sufficient length of the analysed time series has a great influence on the correct results (see e.g. Booth et al., 2005, or Šimpach et al., 2014). There are countries with detailed statistics for long periods of time. On the other hand, there are also states without any, as they either never published them or gathered them because of political, economic or social circumstances that occurred. Other reasons for incomplete data are territorial and political transformations, the establishment of new states and civil and world wars.

The methods of stochastic modelling are described in detail for example by Bell and Monsell (1991). Lee and Carter's (1992) model, further extended by Lee and

[^1]Tuljapurkar (1994), represents a real milestone in stochastic demographic modelling.

Stochastic predictions and the Lee-Carter (LC) models were used and further developed for example by Arltová (2011), who applied stochastic modelling using the Lee-Carter model to the Czech population, Arlt and Arltová (2011), who forecasted mortality using the co-integrated Lee-Carter method, and Šimpach and Langhamrová (2014), who predicted ageand sex-specific mortality rates. ${ }^{1}$ Similarly, D'Amato et al. (2011) used the LC model for forecasting the mortality of the Italian population. The Bayesian and probabilistic approach to population forecasting was incorporated into the LC model by Wiśniowski et al. (2015).

### 1.2 Methodology of migration modelling

The prediction of the migration processes themselves is more complicated. Among the most important constraints of migration research so far are the lack of detailed data, a significant delay in the publication of public data and even the disparity of estimates or expert judgements within the publications of local authorities. From a demographic perspective, the migration process, compared with mortality and fertility, is not yet adequately supported by models. Together with Ediev et al. (2014), we may proclaim that, although migration has become a key factor for growth and renewal of the population, the demographic tools for its analysis remain simple.

Migration processes were examined for example by Arltová and Langhamrová (2010), who investigated migration as a solution to population ageing and decrease. A similar topic was assessed in the context of environmental changes by Harper (2012). Šimpach and Pechrová (2016) analysed the possibilities of using the LC model for migration projection. Our paper continues this effort and applies the LC model to the population in France, Belgium, Germany and Italy, because these countries are very often considered by immigrants as their terminal country.

## 2. Methodology

The article deals with immigration to European countries that are considered to be the target for immigrants from third countries. Specifically we examine the migration to France, Belgium, Germany and Italy. While in other countries (considered as transfer) the immigration problem is not that
pronounced, the selected countries may experience severe changes.

### 2.1 Statistical description of the data

In the first part, the article assesses the evolution of the number of immigrants and emigrants to and from selected EU terminal countries (France, Belgium, Germany and Italy) and the number of immigrants per 1,000 inhabitants in individual states. In the second part, we pay attention to the age- and sex-specific structure of the immigrants in the countries surveyed. The development is displayed in 1-year periods from 2006 to 2014. However, data are not available for all states and years. The subsequent principal component analysis of immigration must be applied to the agespecific rates of immigration by gender (instead of ageand sex-specific numbers of immigrants), because singular value decomposition of the data matrix (in Lee-Carter analysis) cannot be performed.

### 2.2 Lee-Carter model

Similarly to the logarithms of age- and sex-specific mortality rates and age-specific fertility rates, it is possible to apply the Lee-Carter model (Lee and Carter, 1992; Lee and Tuljapurkar, 1994) to the immigration profile. The projections will be made for two years' prediction horizon - for the years 2015 and 2016 - which are not known at present and are very important for current political affairs. Finally, we discuss the problems and challenges involved in modelling the demographic process of immigration. The basic idea of the LC model is implemented in the decomposition of empirical age- and sex-specific immigration rates in appropriate time periods. The model can be written as

$$
\begin{equation*}
i_{x, t}^{M / F}=a_{x}^{M / F}+b_{x}^{M / F} \cdot k_{t}^{M / F}+\varepsilon_{x, t}^{M / F}, \tag{1}
\end{equation*}
$$

- where age $x=0,1,2, \ldots, 100+$ years (according to the Eurostat publication methodology),
- time $t=1,2, \ldots T$,
- parameters $a_{x}{ }^{M / F}$ are the age-specific immigration profiles independent of time,
- $b_{x}{ }^{M / F}$ are the additional age-specific component that determines the change in the level of immigration in each age group when the indicator $k_{t}^{M / F}$ changes,
- $k_{t}^{M / F}$ are time-varying parameters - the total immigration indices,
- $\varepsilon_{x}{ }^{M / F}$ is a residual element with the characteristics of a white-noise process, where mean $E\left(\varepsilon_{x}\right)=0$, variance (dispersion) $D\left(\varepsilon_{x}\right)=\sigma^{2}$, $\operatorname{cov}\left(\varepsilon_{x} ; \varepsilon_{x}{ }^{\prime}\right)=0$ and $\varepsilon_{x} \approx \mathrm{~N}$ distribution,
- $\quad M$ and $F$ denote gender.

The age- and sex-specific rates of immigration are not published in the Eurostat database, so we have to calculate them. We need to know the age- and sex-
specific numbers of immigrants in the particular years $I_{x}{ }^{M / F}(2006-2014$, see Eurostat, 2016) and exposure to risk, which is estimated as the age- and sex-specific mid-year population state in the particular year. Then, using the ratio

$$
\begin{equation*}
i_{x}^{M / F}=\frac{I_{x, t}^{M / F}}{\bar{S}_{x, t}^{M / F}}, \tag{2}
\end{equation*}
$$

where

$$
\begin{equation*}
\bar{S}_{x, t}^{M / F}=\frac{S_{x, t=1.1 . y y y}^{M / F}+S_{x, t=1.1 . y y y+1}^{M / F}}{2} \tag{3}
\end{equation*}
$$

we obtain the age- and sex-specific immigration rates. The estimation of parameters $b_{x}{ }^{M / F}$ and $k_{t}^{M / F}$ is based on the principle of singular value decomposition (SVD) of the matrix of these rates, as presented for example in the case of mortality by Bell and Monsell (1991), Lee and Carter (1992) and Hyndman and Ullah (2010). The age- and sex-specific immigration rates $i_{x}^{M / F}$ at the exact age $x$ and in time $t$ create $x \times T$ dimensional matrix

$$
\begin{equation*}
\mathbf{I}=\mathbf{A}+\mathbf{B} \mathbf{K}^{\mathrm{T}}+\mathbf{E} \tag{4}
\end{equation*}
$$

The identification of the LC model is ensured by restrictive conditions

$$
\begin{equation*}
\sum_{x=0}^{100+} b_{x}^{M / F}=1 \quad \text { and } \quad \sum_{t=1}^{T} k_{t}^{M / F}=0 \tag{5}
\end{equation*}
$$

The simple arithmetic average of age-specific immigration rates by gender is calculated as

$$
\begin{equation*}
a_{x}^{M / F}=\frac{\sum_{t=1}^{T} i_{x, t}^{M / F}}{T} . \tag{6}
\end{equation*}
$$

For the prediction of age- and sex-specific immigration rates, it is necessary to predict the values of parameter $k_{t}^{M / F}$, which is mostly conducted using the ARIMA ( $p, d, q$ ) models.

The data for France, Belgium, Germany and Italy about immigration and emigration in the division according to age and sex in one-year intervals were obtained from the Eurostat (2016) database. Net migration is calculated as

$$
\begin{equation*}
\Delta_{t}^{M / F}=I_{t}^{M / F}-E_{t}^{M / F}, \tag{7}
\end{equation*}
$$

where $I_{t}{ }^{M / F}$ and $E_{t}{ }^{M / F}$ are the total numbers of immigrants and emigrants in particular years and countries.

## 3. Results

The number of immigrants in the observed countries has rapidly changed recently due to the immigration crisis, in which France, Belgium, Germany and Italy are seen as terminal destinations of the refugees. However, the net migration balance of those countries was also positive in the past (immigration exceeded emigration). The only exception is Germany in the year 2008, when 55743 more people emigrated than arrived. The


Figure 1 Immigration, emigration and net migration in selected Member States of the EU
Source: Own elaboration, Eurostat (2016)
development of the immigration, emigration and net migration is displayed in Figure 1.

### 3.1 Immigration and emigration in the EU states

The number of immigrants in France rose continuously from 307111 people in 2010 to 339902 in 2014. In Belgium the situation was different, as 3 waves of increase can be observed there: from 2005 to 2007 (there are no available (N/A) data for 2008 and 2009), from 2010 to 2012 and again from 2013 onwards. A significant decrease happened in Germany between 2008 and 2009, but the number of immigrants has been rising since then (from 346216 in 2009 to 884893 in 2014). In Italy the highest level of immigration occurred in 2007, when 558019 people arrived.

Regarding the gender structure, there were more females than males in France. Belgium and Germany experienced the opposite situation. In Italy there were more females, but in 2014 the number of male immigrants increased and exceeded the number of females.

In France the number of emigrants rose from 2006 (there are no data available for the year 2005) until 2011, the next year the figure was lower and it started to increase again after that. Emigration from Belgium was increasing following the same pattern as the immigration from Belgium in the waves from 2005 to

2007 and from 2010 to 2013 but decreased in 2014. Emigration from Germany was high until 2008, and then it decreased to a low and stabilized level. In Italy the level of emigration was relatively low (under 100 thousand people) until 2011. It started to increase in 2012.

The gender structure of emigrants was similar to that of immigrants. In France the structure was in favour of females; only in the years 2008 and 2011 was the number of males higher. In Belgium, Germany and Italy, more males emigrated than females.

Recalculating the number of immigrants to 1000 inhabitants (see Figure 2), the highest numbers


Figure 2 The number of immigrants in 1000 inhabitants in selected Member States of the EU
Source: Own elaboration, Eurostat (2016)

## France



Figure 3a Age-and-sex specific structures of immigrants to France (different colours mark year)
Source: Own elaboration, Eurostat (2016)

## Germany



Figure 3c Age-and-sex specific structures of immigrants to Germany (different colours mark year)
Source: Own elaboration, Eurostat (2016)
occurred in Belgium throughout the whole period. The situation was the same only in Germany in 2014 (11 immigrants per 1000 inhabitants). The most immigrants in absolute terms arrived in this country (with the exception of 2009 and 2010, when the most arrived in Italy), but their share was not as significant as in Italy (until 2011). In France the share did not change much (around 5 immigrants/1000 inhabitants).

## Belgium



Figure 3b Age-and-sex specific structures of immigrants to Belgium (different colours mark year)
Source: Own elaboration, Eurostat (2016)

## Italy






Figure 3d Age-and-sex specific structures of immigrants to Italy (different colours mark year)
Source: Own elaboration, Eurostat (2016)
From Figure 3 (and Figure 7 in the annex), it can be seen that the most immigrants were aged 21 (males) and 22 (females) in France. In Belgium the peak was 23 for males and 24 for females. German immigrants were also between 20 and 25 years old, and a similar situation occurred in Italy. Besides, there was an interesting local peak for women aged 40.

### 3.2 Immigration predictions for the EU states

Using the singular value decomposition method implemented in the package demography (Hyndman, 2012), which was developed for RStudio (R Development Core Team, 2008), we estimated the parameters $a_{x}$ (the age-specific immigration profiles independent of time) and $b_{x}$ (the additional agespecific component that determines the change in the level of immigration in each age group when the indicator $k_{t}$ changes) for all the Lee-Carter models (4 countries $\times 2$ genders). We can see them in Figure 4. In this figure the comparison between the different evolutions of these parameters, depending on the input variability and data availability, is also shown.

The time-varying parameters - the total immigration indices $k_{t}$ - were also estimated for all eight models, and it was found that the results for Belgium, Germany and Italy are almost identical. This is mainly due to the fact that these countries do not have complete data matrix like France and, because the estimates are a little distorted here, they influence the result of the final prediction. We can also see these estimates in Figure 4. For these estimates we calculated the predictions for the years 2015 and 2016 based on the methodological approach of ARIMA $(p, d, q)$ (Box and Jenkins, 1970) and run by the forecast package in RStudio (by Hyndman et al., 2002, or Hyndman and Shang, 2009).

## France



Figure 4a Estimated parameters $a_{x}, b_{x}$ and $k_{t}$ for France

## Belgium



Figure 4b Estimated parameters $a_{x}, b_{x}$ and $k_{t}$ for Belgium

## Germany



Figure $\mathbf{4 c}$ Estimated parameters $a_{x}, b_{x}$ and $k_{t}$ for Germany

## Italy




Figure 4d Estimated parameters $a_{x}, b_{x}$ and $k_{t}$ for Italy
We must keep in mind that performing predictions from short and variable databases is very difficult. Eurostat does not have longer databases. Therefore, better predictions cannot be calculated (the time series of indices $k_{t}$ is short) and hence the predictions have relatively wide $95 \%$ confidence intervals. This is possible to observe from Figure 5 for each country. Because the indices $k_{t}$ have no deterministic development, it is difficult to assume any future shape or distribution for them. We expect a general increase, which is evident in all the graphs.

## France



Figure 5a Predictions of indices $k_{t}$ for years 2015 and 2016 for France with $95,99 \%$ (dark, light) confidence intervals

## Germany



Figure 5b Predictions of indices $k_{t}$ for years 2015 and 2016 for Germany with 95 , $99 \%$ (dark, light) confidence intervals

## Belgium



Figure 5c Predictions of indices $k_{t}$ for years 2015 and 2016 for Belgium with 95, $99 \%$ (dark, light) confidence intervals

## Italy




Figure 5d Predictions of indices $k_{t}$ for years 2015 and 2016 for Italy with $95,99 \%$ (dark, light) confidence intervals

Based on the estimated parameters $a_{x}, b_{x}$ and $k_{t}$ (and the extrapolation of $k_{t}$ ) of all eight Lee-Carter models (for males and for females in France, Belgium, Germany and Italy), we now fit and then estimate the future values of $i_{x, t}{ }^{M / F}$ for $t=2015$ and 2016. The results in graphical form can be seen in Figure 6, and tabulated values are presented in Table 1-Table 4 in the Annex of the paper. The $y$-axis reports the number of $x$-old immigrants (of the particular gender) in this country per living person (also of the particular gender) and aged $x$ years. We must view the results critically, because only in the case of France do they seem to be relevant. In the case of Belgium, Germany and Italy, the immigration profile is inappropriately high, although it is true that


## Germany



## Belgium



Italy


Figure 6 Estimated age- and sex-specific immigration profiles for the years 2015 and 2016 in France, Belgium, Germany and Italy
the immigration wave was particularly extreme in 2015 and 2016 and the official statistics have not yet been published. The inappropriately high levels in Belgium, Germany and France are caused by deflection due to an insufficiently long database. The model is of high quality only when it is supported by a high-quality data matrix. When we obtain a sufficient data matrix, these rates can be applied to correct the results of the population structure of the country as

$$
\begin{equation*}
I_{x, t}^{M / F}=i_{x, t}^{M / F} \cdot \bar{S}_{x, t}^{M / F}, \tag{8}
\end{equation*}
$$

because the age- and sex-specific number of immigrants can be used in the cohort form of the basic population formula.

## 4. Discussion

In addition to the description of migration issues to selected terminal European Union countries, this article applies the LC model to immigration processes. However, the first aspect of the LC model that has to be considered is that it needs a sufficiently long database that should be as stable as possible (see e.g. Šimpach et al., 2014). Instability in the development of the matrix of immigration time series causes the deflection of the average migration profile $a_{x}{ }^{M / F}$ and thus biases the estimates of future values. To find the main components explaining the trend
and previous development, the database must be sufficiently long.

The second issue is that, because migration has not been such a widely discussed topic until recently, not enough attention has been paid to these statistics and most of the European states lack quality data sets in mutually comparable datasheets (see e.g. Lundström and Qvist (2004), who solved a similar problem in the case of the mortality process in the Swedish population).

The third aspect is related to the model itself. The standard LC model, which uses singular value decomposition, assumes that the errors have a constant variance over all ages (Koissi and Shapiro, 2006).

However, this does not often hold. Therefore, Koissi and Shapiro (2006) suggested a fuzzy approach whereby the errors are viewed as fuzziness of the model structure; hence, the homoscedasticity is not an issue. Only when particular European countries, which are affected the most by high immigration levels, update their databases into a uniform and comparable form can the modified approach of Koissi and Shapiro (2006) be applied, as the correction suggested by them is not susceptible to insufficient length of the analysed time series. In addition, the Lee-Carter approach, which was presented in this paper, will be applicable in the
future. The prediction of the immigration profile in France, which seems to be the only relevant one among all the predictions made in this paper, can serve as proof of this possibility.

The fourth issue is related to the migration process, which is of a partly random and partly systematic nature. Classical methods of time series when future developments are projected based on the past development are not sufficient. It is necessary to consider more advanced models that will have to be determined either by the components that explain the migration or by additional explanatory variables. In the process of migration, there are fixed patterns for projection, as in the case of mortality rates and (after the acceptance of certain restrictions) fertility rates.

Despite the fact that migration is an important variable in population projections, it is frequently assumed (especially for small territorial units such as cities, counties and other territorial administrative units), that the migration balance is zero. In cases when a positive or negative value of migration is set based on expert expectations, it is usually corrected by the gender-specific, not the age-specific, population.

## 5. Conclusion

Although migration is currently a much-discussed issue, its modelling is still difficult, especially due to the lack of long time series. The aim of the paper was to discuss the current immigration wave to the European Union and to foresee the impacts of this wave on demographic modelling tools. We applied the stochastic Lee-Carter model originally designed for logarithms of mortality rates and for fertility rates. Therefore, there are certain limitations in applying this method to immigration processes. The results seem to be relevant only in the case of France. In Belgium, Germany and Italy, the immigration profile is inappropriately high. Some of the statistical conditions were violated. The time series were too short and the confidence intervals too wide. When a longer database is available, we suggest further development and use of the Lee-Carter model to project migration processes.

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Annex

## France



Figure 7a Age- and sex-specific structures of immigrants to France over time (different colours mark ages)
Source: Own elaboration, Eurostat (2016)

## Belgium



Figure 7b Age- and sex-specific structures of immigrants to Belgium over time (different colours mark ages)
Source: Own elaboration, Eurostat (2016)

## Germany



Figure 7c Age- and sex-specific structures of immigrants to Germany over time (different colours mark ages)
Source: Own elaboration, Eurostat (2016)

## Italy



Figure 7d Age- and sex-specific structures of immigrants to Italy over time (different colours mark ages)
Source: Own elaboration, Eurostat (2016)

Table 1 Estimated age- and sex-specific immigration rates for males and females in France, years 2015 and 2016

| Age | $F R-M-2015$ | $F R-M-2016$ | $F R-F-2015$ | $F R-F-2016$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0.012 | 0.012 | 0.012 | 0.013 | 50 | 0.003 | 0.003 | 0.003 | 0.003 |
| 1 | 0.006 | 0.006 | 0.006 | 0.006 | 51 | 0.003 | 0.003 | 0.002 | 0.002 |
| 2 | 0.006 | 0.006 | 0.006 | 0.006 | 52 | 0.003 | 0.003 | 0.002 | 0.002 |
| 3 | 0.006 | 0.006 | 0.006 | 0.006 | 53 | 0.002 | 0.002 | 0.002 | 0.002 |
| 4 | 0.005 | 0.005 | 0.006 | 0.006 | 54 | 0.002 | 0.002 | 0.002 | 0.002 |
| 5 | 0.005 | 0.006 | 0.005 | 0.005 | 55 | 0.002 | 0.002 | 0.002 | 0.002 |
| 6 | 0.005 | 0.005 | 0.005 | 0.005 | 56 | 0.002 | 0.002 | 0.002 | 0.002 |
| 7 | 0.004 | 0.004 | 0.005 | 0.005 | 57 | 0.002 | 0.002 | 0.002 | 0.002 |
| 8 | 0.005 | 0.005 | 0.005 | 0.005 | 58 | 0.002 | 0.002 | 0.002 | 0.002 |
| 9 | 0.005 | 0.005 | 0.004 | 0.004 | 59 | 0.001 | 0.001 | 0.001 | 0.001 |
| 10 | 0.004 | 0.004 | 0.004 | 0.004 | 60 | 0.002 | 0.002 | 0.001 | 0.001 |
| 11 | 0.005 | 0.005 | 0.004 | 0.004 | 61 | 0.002 | 0.002 | 0.001 | 0.001 |
| 12 | 0.004 | 0.004 | 0.004 | 0.004 | 62 | 0.002 | 0.002 | 0.001 | 0.001 |
| 13 | 0.004 | 0.004 | 0.004 | 0.004 | 63 | 0.001 | 0.001 | 0.001 | 0.001 |
| 14 | 0.004 | 0.004 | 0.004 | 0.004 | 64 | 0.001 | 0.001 | 0.001 | 0.001 |
| 15 | 0.005 | 0.005 | 0.005 | 0.005 | 65 | 0.001 | 0.001 | 0.001 | 0.001 |
| 16 | 0.006 | 0.006 | 0.004 | 0.004 | 66 | 0.002 | 0.002 | 0.001 | 0.001 |
| 17 | 0.006 | 0.006 | 0.004 | 0.004 | 67 | 0.001 | 0.001 | 0.001 | 0.001 |
| 18 | 0.010 | 0.010 | 0.012 | 0.012 | 68 | 0.001 | 0.001 | 0.001 | 0.001 |
| 19 | 0.008 | 0.008 | 0.012 | 0.012 | 69 | 0.001 | 0.001 | 0.001 | 0.001 |
| 20 | 0.010 | 0.010 | 0.015 | 0.015 | 70 | 0.001 | 0.001 | 0.001 | 0.001 |
| 21 | 0.014 | 0.014 | 0.021 | 0.021 | 71 | 0.001 | 0.001 | 0.001 | 0.001 |
| 22 | 0.016 | 0.016 | 0.024 | 0.024 | 72 | 0.001 | 0.001 | 0.001 | 0.001 |
| 23 | 0.017 | 0.017 | 0.024 | 0.024 | 73 | 0.001 | 0.001 | 0.001 | 0.001 |
| 24 | 0.016 | 0.016 | 0.020 | 0.020 | 74 | 0.001 | 0.001 | 0.001 | 0.000 |
| 25 | 0.015 | 0.015 | 0.018 | 0.018 | 75 | 0.001 | 0.001 | 0.001 | 0.001 |
| 26 | 0.014 | 0.014 | 0.016 | 0.016 | 76 | 0.001 | 0.001 | 0.001 | 0.001 |
| 27 | 0.012 | 0.012 | 0.013 | 0.013 | 77 | 0.000 | 0.000 | 0.001 | 0.001 |
| 28 | 0.011 | 0.011 | 0.013 | 0.013 | 78 | 0.001 | 0.001 | 0.000 | 0.000 |
| 29 | 0.011 | 0.010 | 0.012 | 0.012 | 79 | 0.000 | 0.000 | 0.000 | 0.000 |
| 30 | 0.011 | 0.011 | 0.010 | 0.010 | 80 | 0.000 | 0.000 | 0.000 | 0.000 |
| 31 | 0.009 | 0.010 | 0.010 | 0.010 | 81 | 0.000 | 0.000 | 0.000 | 0.000 |
| 32 | 0.009 | 0.009 | 0.009 | 0.010 | 82 | 0.000 | 0.000 | 0.000 | 0.000 |
| 33 | 0.008 | 0.008 | 0.009 | 0.009 | 83 | 0.000 | 0.000 | 0.000 | 0.000 |
| 34 | 0.008 | 0.008 | 0.008 | 0.008 | 84 | 0.000 | 0.000 | 0.000 | 0.000 |
| 35 | 0.007 | 0.007 | 0.008 | 0.008 | 85 | 0.000 | 0.000 | 0.000 | 0.000 |
| 36 | 0.007 | 0.007 | 0.008 | 0.008 | 86 | 0.001 | 0.001 | 0.000 | 0.000 |
| 37 | 0.006 | 0.006 | 0.007 | 0.007 | 87 | 0.001 | 0.001 | 0.000 | 0.000 |
| 38 | 0.006 | 0.006 | 0.006 | 0.006 | 88 | 0.001 | 0.001 | 0.000 | 0.000 |
| 39 | 0.005 | 0.005 | 0.006 | 0.006 | 89 | 0.000 | 0.000 | 0.000 | 0.000 |
| 40 | 0.005 | 0.005 | 0.005 | 0.005 | 90 | 0.000 | 0.000 | 0.000 | 0.000 |
| 41 | 0.005 | 0.005 | 0.005 | 0.005 | 91 | 0.000 | 0.000 | 0.001 | 0.001 |
| 42 | 0.004 | 0.004 | 0.004 | 0.004 | 92 | 0.000 | 0.000 | 0.000 | 0.000 |
| 43 | 0.005 | 0.005 | 0.004 | 0.004 | 93 | 0.000 | 0.000 | 0.000 | 0.000 |
| 44 | 0.004 | 0.004 | 0.004 | 0.004 | 94 | 0.000 | 0.000 | 0.000 | 0.000 |
| 45 | 0.004 | 0.004 | 0.004 | 0.004 | 95 | 0.001 | 0.001 | 0.000 | 0.000 |
| 46 | 0.004 | 0.005 | 0.003 | 0.003 | 96 | 0.001 | 0.002 | 0.000 | 0.000 |
| 47 | 0.004 | 0.004 | 0.003 | 0.003 | 97 | 0.002 | 0.006 | 0.000 | 0.000 |
| 48 | 0.003 | 0.003 | 0.003 | 0.003 | 98 | 0.000 | 0.000 | 0.000 | 0.000 |
| 49 | 0.003 | 0.003 | 0.003 | 0.003 | 99 | 0.000 | 0.000 | 0.000 | 0.000 |
| $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | 100+ | 0.000 | 0.000 | 0.000 | 0.000 |

Table 2 Estimated age- and sex-specific immigration rates for males and females in Belgium, years 2015 and 2016

| Age | BE-M-2015 | BE-M-2016 | BE - F-2015 | BE-F-2016 | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0.096 | 0.739 | 0.099 | 0.765 | 50 | 0.056 | 0.405 | 0.038 | 0.267 |
| 1 | 0.126 | 1.001 | 0.126 | 1.001 | 51 | 0.049 | 0.347 | 0.035 | 0.245 |
| 2 | 0.117 | 0.919 | 0.116 | 0.915 | 52 | 0.048 | 0.340 | 0.032 | 0.222 |
| 3 | 0.108 | 0.846 | 0.107 | 0.832 | 53 | 0.044 | 0.312 | 0.029 | 0.199 |
| 4 | 0.102 | 0.795 | 0.101 | 0.784 | 54 | 0.039 | 0.275 | 0.028 | 0.187 |
| 5 | 0.095 | 0.733 | 0.098 | 0.759 | 55 | 0.037 | 0.253 | 0.027 | 0.184 |
| 6 | 0.095 | 0.735 | 0.095 | 0.729 | 56 | 0.033 | 0.226 | 0.025 | 0.169 |
| 7 | 0.089 | 0.677 | 0.087 | 0.661 | 57 | 0.032 | 0.216 | 0.023 | 0.153 |
| 8 | 0.087 | 0.659 | 0.087 | 0.664 | 58 | 0.028 | 0.186 | 0.021 | 0.138 |
| 9 | 0.079 | 0.597 | 0.082 | 0.622 | 59 | 0.025 | 0.164 | 0.021 | 0.135 |
| 10 | 0.080 | 0.603 | 0.080 | 0.604 | 60 | 0.023 | 0.149 | 0.018 | 0.114 |
| 11 | 0.076 | 0.567 | 0.077 | 0.580 | 61 | 0.021 | 0.134 | 0.018 | 0.112 |
| 12 | 0.074 | 0.555 | 0.074 | 0.550 | 62 | 0.019 | 0.125 | 0.015 | 0.091 |
| 13 | 0.072 | 0.538 | 0.069 | 0.513 | 63 | 0.017 | 0.110 | 0.014 | 0.085 |
| 14 | 0.068 | 0.504 | 0.068 | 0.501 | 64 | 0.016 | 0.099 | 0.013 | 0.083 |
| 15 | 0.068 | 0.506 | 0.065 | 0.480 | 65 | 0.020 | 0.132 | 0.013 | 0.081 |
| 16 | 0.086 | 0.652 | 0.070 | 0.520 | 66 | 0.015 | 0.093 | 0.012 | 0.073 |
| 17 | 0.089 | 0.681 | 0.073 | 0.542 | 67 | 0.014 | 0.089 | 0.010 | 0.059 |
| 18 | 0.104 | 0.813 | 0.120 | 0.949 | 68 | 0.012 | 0.071 | 0.011 | 0.069 |
| 19 | 0.115 | 0.908 | 0.149 | 1.204 | 69 | 0.010 | 0.060 | 0.009 | 0.051 |
| 20 | 0.140 | 1.128 | 0.168 | 1.379 | 70 | 0.011 | 0.064 | 0.010 | 0.060 |
| 21 | 0.151 | 1.227 | 0.198 | 1.655 | 71 | 0.009 | 0.056 | 0.008 | 0.049 |
| 22 | 0.181 | 1.493 | 0.233 | 1.985 | 72 | 0.008 | 0.048 | 0.008 | 0.044 |
| 23 | 0.212 | 1.786 | 0.259 | 2.230 | 73 | 0.008 | 0.046 | 0.007 | 0.042 |
| 24 | 0.246 | 2.101 | 0.278 | 2.408 | 74 | 0.006 | 0.035 | 0.006 | 0.035 |
| 25 | 0.275 | 2.379 | 0.283 | 2.464 | 75 | 0.006 | 0.037 | 0.006 | 0.033 |
| 26 | 0.279 | 2.419 | 0.271 | 2.343 | 76 | 0.006 | 0.034 | 0.006 | 0.031 |
| 27 | 0.278 | 2.410 | 0.255 | 2.196 | 77 | 0.006 | 0.033 | 0.006 | 0.032 |
| 28 | 0.265 | 2.283 | 0.232 | 1.974 | 78 | 0.005 | 0.026 | 0.004 | 0.023 |
| 29 | 0.253 | 2.172 | 0.215 | 1.812 | 79 | 0.005 | 0.028 | 0.006 | 0.032 |
| 30 | 0.242 | 2.063 | 0.194 | 1.613 | 80 | 0.005 | 0.027 | 0.004 | 0.023 |
| 31 | 0.222 | 1.881 | 0.181 | 1.500 | 81 | 0.004 | 0.022 | 0.004 | 0.024 |
| 32 | 0.209 | 1.753 | 0.165 | 1.350 | 82 | 0.004 | 0.021 | 0.004 | 0.022 |
| 33 | 0.198 | 1.652 | 0.155 | 1.264 | 83 | 0.004 | 0.023 | 0.004 | 0.021 |
| 34 | 0.190 | 1.578 | 0.143 | 1.152 | 84 | 0.004 | 0.021 | 0.004 | 0.024 |
| 35 | 0.178 | 1.467 | 0.137 | 1.095 | 85 | 0.004 | 0.024 | 0.004 | 0.023 |
| 36 | 0.169 | 1.387 | 0.124 | 0.984 | 86 | 0.005 | 0.027 | 0.005 | 0.030 |
| 37 | 0.155 | 1.259 | 0.116 | 0.915 | 87 | 0.004 | 0.022 | 0.006 | 0.031 |
| 38 | 0.143 | 1.149 | 0.103 | 0.797 | 88 | 0.007 | 0.038 | 0.005 | 0.026 |
| 39 | 0.130 | 1.037 | 0.094 | 0.724 | 89 | 0.004 | 0.021 | 0.006 | 0.034 |
| 40 | 0.125 | 0.989 | 0.085 | 0.647 | 90 | 0.004 | 0.024 | 0.004 | 0.022 |
| 41 | 0.114 | 0.895 | 0.080 | 0.608 | 91 | 0.004 | 0.024 | 0.005 | 0.030 |
| 42 | 0.105 | 0.815 | 0.072 | 0.538 | 92 | 0.003 | 0.015 | 0.005 | 0.030 |
| 43 | 0.100 | 0.773 | 0.064 | 0.475 | 93 | 0.000 | 0.002 | 0.004 | 0.023 |
| 44 | 0.089 | 0.681 | 0.059 | 0.427 | 94 | 0.000 | 0.001 | 0.007 | 0.040 |
| 45 | 0.085 | 0.647 | 0.055 | 0.398 | 95 | 0.000 | 0.000 | 0.005 | 0.027 |
| 46 | 0.075 | 0.561 | 0.051 | 0.369 | 96 | 0.000 | 0.000 | 0.006 | 0.033 |
| 47 | 0.070 | 0.525 | 0.046 | 0.323 | 97 | 0.000 | 0.000 | 0.000 | 0.001 |
| 48 | 0.065 | 0.476 | 0.043 | 0.301 | 98 | 0.000 | 0.000 | 0.005 | 0.030 |
| 49 | 0.058 | 0.420 | 0.040 | 0.282 | 99 | 0.000 | 0.000 | 0.000 | 0.000 |
| $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | 100+ | 0.000 | 0.000 | 0.000 | 0.000 |

Table 3 Estimated age- and sex-specific immigration rates for males and females in Germany, years 2015 and 2016

| Age | GE-M-2015 | GE-M-2016 | GE - F-2015 | GE-F-2016 | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0.107 | 0.832 | 0.096 | 0.740 | 50 | 0.057 | 0.416 | 0.030 | 0.203 |
| 1 | 0.152 | 1.229 | 0.136 | 1.086 | 51 | 0.054 | 0.388 | 0.029 | 0.192 |
| 2 | 0.129 | 1.030 | 0.114 | 0.899 | 52 | 0.051 | 0.369 | 0.027 | 0.183 |
| 3 | 0.116 | 0.911 | 0.106 | 0.824 | 53 | 0.048 | 0.342 | 0.026 | 0.177 |
| 4 | 0.105 | 0.820 | 0.096 | 0.738 | 54 | 0.045 | 0.317 | 0.025 | 0.164 |
| 5 | 0.100 | 0.773 | 0.091 | 0.698 | 55 | 0.041 | 0.287 | 0.023 | 0.150 |
| 6 | 0.091 | 0.699 | 0.082 | 0.622 | 56 | 0.037 | 0.257 | 0.021 | 0.138 |
| 7 | 0.080 | 0.606 | 0.074 | 0.558 | 57 | 0.033 | 0.229 | 0.019 | 0.121 |
| 8 | 0.075 | 0.559 | 0.068 | 0.508 | 58 | 0.028 | 0.187 | 0.017 | 0.109 |
| 9 | 0.072 | 0.538 | 0.065 | 0.479 | 59 | 0.025 | 0.167 | 0.016 | 0.098 |
| 10 | 0.067 | 0.495 | 0.061 | 0.446 | 60 | 0.023 | 0.149 | 0.015 | 0.096 |
| 11 | 0.062 | 0.453 | 0.054 | 0.393 | 61 | 0.020 | 0.129 | 0.014 | 0.084 |
| 12 | 0.058 | 0.422 | 0.052 | 0.374 | 62 | 0.019 | 0.120 | 0.013 | 0.078 |
| 13 | 0.057 | 0.413 | 0.049 | 0.353 | 63 | 0.017 | 0.106 | 0.012 | 0.075 |
| 14 | 0.056 | 0.409 | 0.047 | 0.336 | 64 | 0.015 | 0.097 | 0.011 | 0.069 |
| 15 | 0.063 | 0.466 | 0.048 | 0.345 | 65 | 0.018 | 0.112 | 0.011 | 0.069 |
| 16 | 0.091 | 0.696 | 0.061 | 0.447 | 66 | 0.013 | 0.082 | 0.009 | 0.055 |
| 17 | 0.087 | 0.667 | 0.060 | 0.436 | 67 | 0.011 | 0.069 | 0.008 | 0.044 |
| 18 | 0.141 | 1.139 | 0.108 | 0.840 | 68 | 0.010 | 0.060 | 0.007 | 0.038 |
| 19 | 0.219 | 1.852 | 0.186 | 1.546 | 69 | 0.009 | 0.053 | 0.006 | 0.033 |
| 20 | 0.238 | 2.031 | 0.224 | 1.898 | 70 | 0.008 | 0.044 | 0.005 | 0.030 |
| 21 | 0.273 | 2.361 | 0.249 | 2.131 | 71 | 0.007 | 0.037 | 0.005 | 0.026 |
| 22 | 0.309 | 2.715 | 0.277 | 2.406 | 72 | 0.006 | 0.036 | 0.005 | 0.025 |
| 23 | 0.343 | 3.042 | 0.287 | 2.499 | 73 | 0.006 | 0.031 | 0.004 | 0.021 |
| 24 | 0.348 | 3.098 | 0.270 | 2.336 | 74 | 0.005 | 0.029 | 0.004 | 0.020 |
| 25 | 0.339 | 3.003 | 0.254 | 2.176 | 75 | 0.005 | 0.027 | 0.004 | 0.019 |
| 26 | 0.331 | 2.927 | 0.231 | 1.958 | 76 | 0.004 | 0.023 | 0.004 | 0.019 |
| 27 | 0.314 | 2.763 | 0.211 | 1.777 | 77 | 0.004 | 0.022 | 0.003 | 0.018 |
| 28 | 0.303 | 2.657 | 0.196 | 1.638 | 78 | 0.004 | 0.021 | 0.003 | 0.018 |
| 29 | 0.287 | 2.496 | 0.182 | 1.505 | 79 | 0.004 | 0.020 | 0.003 | 0.017 |
| 30 | 0.279 | 2.420 | 0.181 | 1.494 | 80 | 0.003 | 0.019 | 0.003 | 0.016 |
| 31 | 0.266 | 2.297 | 0.165 | 1.353 | 81 | 0.003 | 0.016 | 0.003 | 0.015 |
| 32 | 0.252 | 2.161 | 0.154 | 1.250 | 82 | 0.003 | 0.015 | 0.003 | 0.014 |
| 33 | 0.243 | 2.081 | 0.142 | 1.141 | 83 | 0.003 | 0.015 | 0.003 | 0.014 |
| 34 | 0.233 | 1.981 | 0.133 | 1.059 | 84 | 0.003 | 0.015 | 0.003 | 0.013 |
| 35 | 0.218 | 1.837 | 0.123 | 0.972 | 85 | 0.000 | 0.000 | 0.000 | 0.000 |
| 36 | 0.204 | 1.711 | 0.111 | 0.868 | 86 | 0.001 | 0.001 | 0.001 | 0.001 |
| 37 | 0.187 | 1.556 | 0.099 | 0.769 | 87 | 0.001 | 0.001 | 0.001 | 0.001 |
| 38 | 0.170 | 1.395 | 0.090 | 0.686 | 88 | 0.001 | 0.001 | 0.001 | 0.001 |
| 39 | 0.152 | 1.231 | 0.080 | 0.604 | 89 | 0.000 | 0.000 | 0.000 | 0.000 |
| 40 | 0.135 | 1.077 | 0.071 | 0.529 | 90 | 0.000 | 0.000 | 0.000 | 0.000 |
| 41 | 0.122 | 0.962 | 0.063 | 0.461 | 91 | 0.000 | 0.000 | 0.000 | 0.000 |
| 42 | 0.106 | 0.825 | 0.054 | 0.391 | 92 | 0.000 | 0.000 | 0.000 | 0.000 |
| 43 | 0.096 | 0.742 | 0.049 | 0.352 | 93 | 0.000 | 0.000 | 0.000 | 0.000 |
| 44 | 0.088 | 0.673 | 0.045 | 0.318 | 94 | 0.000 | 0.000 | 0.000 | 0.000 |
| 45 | 0.080 | 0.605 | 0.042 | 0.292 | 95 | 0.001 | 0.001 | 0.001 | 0.001 |
| 46 | 0.074 | 0.555 | 0.038 | 0.266 | 96 | 0.001 | 0.002 | 0.001 | 0.002 |
| 47 | 0.069 | 0.509 | 0.035 | 0.238 | 97 | 0.002 | 0.006 | 0.002 | 0.006 |
| 48 | 0.064 | 0.470 | 0.033 | 0.227 | 98 | 0.000 | 0.000 | 0.000 | 0.000 |
| 49 | 0.061 | 0.447 | 0.032 | 0.215 | 99 | 0.000 | 0.000 | 0.000 | 0.000 |
| $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | 100+ | 0.000 | 0.000 | 0.000 | 0.000 |

Table 4 Estimated age- and sex-specific immigration rates for males and females in Italy, years 2015 and 2016

| Age | IT - M - 2015 | IT-M-2016 | $I T-F-2015$ | IT - F-2016 | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0.015 | 0.097 | 0.014 | 0.088 | 50 | 0.013 | 0.082 | 0.026 | 0.173 |
| 1 | 0.035 | 0.238 | 0.031 | 0.214 | 51 | 0.013 | 0.079 | 0.026 | 0.173 |
| 2 | 0.041 | 0.284 | 0.034 | 0.232 | 52 | 0.012 | 0.070 | 0.026 | 0.173 |
| 3 | 0.039 | 0.274 | 0.033 | 0.222 | 53 | 0.011 | 0.064 | 0.025 | 0.167 |
| 4 | 0.036 | 0.251 | 0.031 | 0.209 | 54 | 0.010 | 0.061 | 0.024 | 0.157 |
| 5 | 0.036 | 0.246 | 0.031 | 0.207 | 55 | 0.009 | 0.055 | 0.023 | 0.153 |
| 6 | 0.035 | 0.240 | 0.030 | 0.203 | 56 | 0.009 | 0.053 | 0.021 | 0.140 |
| 7 | 0.034 | 0.232 | 0.029 | 0.195 | 57 | 0.009 | 0.051 | 0.020 | 0.127 |
| 8 | 0.033 | 0.225 | 0.028 | 0.187 | 58 | 0.008 | 0.047 | 0.018 | 0.117 |
| 9 | 0.033 | 0.227 | 0.029 | 0.195 | 59 | 0.007 | 0.043 | 0.017 | 0.106 |
| 10 <br> 11 | 0.033 | 0.229 | 0.029 | 0.194 | 60 | 0.008 | 0.046 | 0.015 | 0.091 |
| 11 | 0.034 | 0.232 | 0.029 | 0.195 | 61 | 0.007 | 0.042 | 0.013 | 0.080 |
| 12 | 0.034 | 0.232 | 0.029 | 0.193 | 62 | 0.007 | 0.042 | 0.011 | 0.069 |
| 13 | 0.034 | 0.230 | 0.028 | 0.191 | 63 | 0.007 | 0.044 | 0.010 | 0.063 |
| 14 | 0.035 | 0.242 | 0.028 | 0.191 | 64 | 0.007 | 0.041 | 0.010 | 0.059 |
| 15 | 0.037 | 0.257 | 0.028 | 0.190 | 65 | 0.009 | 0.056 | 0.009 | 0.056 |
| 16 <br> 17 | 0.040 | 0.281 | 0.028 | 0.191 | 66 | 0.008 | 0.047 | 0.008 | 0.047 |
| 17 | 0.051 | 0.365 | 0.030 | 0.205 | 67 | 0.007 | 0.038 | 0.007 | 0.041 |
| 18 | 0.076 | 0.572 | 0.045 | 0.321 | 68 | 0.006 | 0.034 | 0.006 | 0.036 |
| 19 | 0.074 | 0.551 | 0.065 | 0.481 | 69 | 0.006 | 0.033 | 0.006 | 0.032 |
| 20 | 0.069 | 0.510 | 0.076 | 0.573 | 70 | 0.005 | 0.029 | 0.005 | 0.028 |
| 21 | 0.074 | 0.558 | 0.085 | 0.649 | 71 | 0.005 | 0.027 | 0.004 | 0.024 |
| 22 | 0.086 | 0.651 | 0.096 | 0.742 | 72 | 0.005 | 0.025 | 0.004 | 0.023 |
| 23 | 0.098 | 0.757 | 0.108 | 0.843 | 73 | 0.004 | 0.022 | 0.004 | 0.021 |
| 24 | 0.111 | 0.867 | 0.113 | 0.889 | 74 | 0.004 | 0.022 | 0.003 | 0.018 |
| 25 | 0.118 | 0.927 | 0.115 | 0.904 | 75 | 0.004 | 0.020 | 0.003 | 0.017 |
| 26 | 0.124 | 0.979 | 0.113 | 0.883 | 76 | 0.004 | 0.020 | 0.003 | 0.015 |
| 27 | 0.123 | 0.978 | 0.106 | 0.827 | 77 | 0.003 | 0.016 | 0.002 | 0.012 |
| 28 | 0.119 | 0.942 | 0.096 | 0.744 | 78 | 0.003 | 0.015 | 0.002 | 0.011 |
| 29 | 0.113 | 0.887 | 0.088 | 0.672 | 79 | 0.003 | 0.014 | 0.002 | 0.010 |
| 30 | 0.105 | 0.813 | 0.080 | 0.600 | 80 | 0.002 | 0.013 | 0.002 | 0.008 |
| 31 | 0.096 | 0.736 | 0.071 | 0.532 | 81 | 0.002 | 0.012 | 0.002 | 0.007 |
| 32 | 0.085 | 0.650 | 0.064 | 0.474 | 82 | 0.002 | 0.010 | 0.001 | 0.007 |
| 33 | 0.077 | 0.575 | 0.058 | 0.421 | 83 | 0.002 | 0.010 | 0.001 | 0.006 |
| 34 | 0.068 | 0.503 | 0.053 | 0.381 | 84 | 0.002 | 0.011 | 0.001 | 0.007 |
| 35 | 0.059 | 0.428 | 0.047 | 0.338 | 85 | 0.002 | 0.009 | 0.001 | 0.006 |
| 36 | 0.053 | 0.382 | 0.043 | 0.306 | 86 | 0.002 | 0.009 | 0.001 | 0.005 |
| 37 | 0.046 | 0.326 | 0.041 | 0.284 | 87 | 0.002 | 0.008 | 0.001 | 0.005 |
| 38 | 0.042 | 0.293 | 0.038 | 0.268 | 88 | 0.002 | 0.008 | 0.001 | 0.004 |
| 39 | 0.037 | 0.258 | 0.037 | 0.256 | 89 | 0.002 | 0.008 | 0.001 | 0.004 |
| 40 | 0.034 | 0.235 | 0.036 | 0.247 | 90 | 0.002 | 0.008 | 0.001 | 0.004 |
| 41 | 0.031 | 0.208 | 0.034 | 0.231 | 91 | 0.001 | 0.007 | 0.001 | 0.004 |
| 42 | 0.027 | 0.182 | 0.032 | 0.217 | 92 | 0.002 | 0.008 | 0.001 | 0.003 |
| 43 | 0.024 | 0.156 | 0.030 | 0.206 | 93 | 0.001 | 0.005 | 0.001 | 0.005 |
| 44 | 0.022 | 0.146 | 0.029 | 0.199 | 94 | 0.001 | 0.006 | 0.001 | 0.004 |
| 45 | 0.020 | 0.127 | 0.028 | 0.188 | 95 | 0.002 | 0.008 | 0.001 | 0.002 |
| 46 | 0.018 | 0.114 | 0.027 | 0.181 | 96 | 0.000 | 0.000 | 0.000 | 0.000 |
| 47 | 0.017 | 0.105 | 0.026 | 0.177 | 97 | 0.000 | 0.001 | 0.001 | 0.002 |
| 48 | 0.015 | 0.094 | 0.026 | 0.173 | 98 | 0.000 | 0.000 | 0.000 | 0.000 |
| 49 | 0.014 | 0.088 | 0.026 | 0.174 | 99 | 0.000 | 0.000 | 0.000 | 0.000 |
| $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | 100+ | 0.000 | 0.000 | 0.000 | 0.000 |


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[^1]:    ${ }^{1}$ The main statistical tool of LC is least-squares estimation via singular value decomposition of the matrix of the log agespecific observed death rates (D'Amato et al., 2011).

