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# Company valuation under interaction in discrete time (real game options model)

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**Abstract**

Company valuation is a crucial topic in financial decision making. The advanced valuation method is the real options approach realised under risk and flexibility. It reflects a stochastic feature of the underlying asset and dynamic managerial decision making. Another aspect of valuation, which is often neglected, is interaction, meaning the mutual impact of other companies on the calculated value. Game theory models this aspect. The paper's objective is to describe and apply company two-phase real game options valuation in discrete time. A generalised real game options valuation model based on the two-phase method, discrete time, risk-neutral probability, and switching cost is formulated. The game categorisation is introduced, especially market structure games, including equilibrium calculations following pure and mixed strategies, and the real game options model is formulated. A company two-phase valuation method in the Cournot production duopoly market structure under random demand is developed, and an illustrative example is presented. The paper confirms the possibility of modelling company two-phase value through real game options valuation models. Neglecting an interaction under a non-perfect market structure can undervalue a company, so this aspect is essential.

**Keywords**

Game theory, real game option, real option, valuation

**JEL Classification:** C7, D4, G13, G32, L1

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# Company valuation under interaction in discrete time (real game options model)

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## Introduction

Company valuation is an important problem in financial decision making and management. The choice of valuation method depends on various aspects. The practical valuation approach is a two-phase discounted cash flow method. The complexity of valuation leads to the frequent application of discrete binomial models. Due to the valuation environment, risk, flexibility, and interaction are substantial aspects. Term risk characterises randomness (stochastic process), flexibility represents a dynamic decision (option valuation), and interactivity means that one company's decision is influenced by other companies' decisions (game theory). The real options method is applied under risk and flexibility. However, interactivity is often neglected, even though it represents a significant aspect of a company's valuation, supposing a non-perfect market.

Researchers and practitioners deal with game real options. Many aspects concerning goals, formulations, methodological conceptions, and application possibilities are investigated (see e.g. Azevedo and Paxson, 2014; Chevalier-Roignant and Trigeorgis, 2011; Grenadier, 2000a, 2000b, 2002; Huisman, 2001; Huisman et al., 2004; Smit and Trigeorgis, 2004, 2017). The problem can be formulated in discrete or continuous time using Bellman's dynamic programming principle. Various optimisation techniques can be applied, game types investigated, the project NPV or value of the company computed, and problems of various sectors investigated and analysed.

The paper focuses on the real game options valuation of a company using the two-phase method in discrete time. This approach has not yet been described in depth in the literature. The paper's objective is to describe and apply company two-phase real game options valuation in discrete time.

Primarily, an interaction feature is investigated. The two-phase discounted cash flow is applied, and the binomial model in discrete time is assumed. The first section is devoted to the development of valuation methods and a description of the real options valuation principles. The chosen aspects of game theory are then described and analysed. Subsequently, the methods of game real options are outlined, especially market structure games. The application of the valuation model with random demand and a duopoly market is verified.

## 1. Valuation methods under risk and flexibility (real options)

The value of company $V$ can be stated through the discounted cash flow method as the present value of cash flow $V=\sum_{t=1}^{\infty} FCF_t \cdot (1+R)^{-t}$ . In the case of the two-phase method

$$V = V1 + V2 = \sum_{t=1}^{T-1} FCF_t \cdot (1+R)^{-t} + V_T \cdot (1+R)^{-T} , \quad (1)$$

$V1$ is the value of the first phase, $V2$ is the value of the second phase, $FCF$ is the free cash flow, $R$ is the risk-free rate, and $V_T$ is the terminal (continual) value, being a value at the beginning of the second phase. Thus, $FCF = EAT + DEP - \Delta NWC - INV$ and, for example, constant perpetuity $V_T = FCF_{T+1} / R$ or growing perpetuity $V_T = FCF_{T+1} / (R-g)$. Then, the present value can be formulated as follows:

$$V = \left\{ \begin{array}{l} \left[ \begin{array}{l} \left( V_T \cdot (1+R)^{-1} + FCF_{T-1} \right) \\ \cdot (1+R)^{-1} + FCF_{T-2} \end{array} \right] \cdot \\ (1+R)^{-1} + ...... + FCF_1 \end{array} \right\} \cdot (1+R)^{-1} .$$

This implies that the recurrent formula for one step is

$$V_t = FCF_t + V_{t+1} \cdot (1+R)^{-1} , \quad (2)$$

and the value equals the free cash flow plus the present value of the one-step future value. The value can be calculated through the backward induction and dynamic programming (Bellman's equation) method.

Valuation under risk means that the underlying asset (factor) is a random process. In a discrete binomial model, the value can be stated using risk-neutral valuation and derived from the replication strategy.

The basic idea is to create a portfolio value $\Pi$ from the underlying free cash flow (asset) $FCF$ and risk-free asset $B$ so a derivative value $V$ can be replicated. For the portfolio value at time $t$, state $i$ is

$$\Pi_t \equiv a \cdot FCF_{i,t} + B_t = V_{i,t} ,$$

the portfolio value at time $t+1$ in upward movement $i+1$ is

$$\Pi_{i+1,t+1} \equiv a \cdot FCF_{i+1,t+1} + B_t \cdot (1+R) = V_{i+1,t+1},$$

the portfolio value at time $t+1$ in downward movement $i-1$ is

$$\Pi_{i-1,t+1} \equiv a \cdot FCF_{i-1,t+1} + B_t \cdot (1+R) = V_{i-1,t+1},$$

and here symbol *a* is the underlying asset quantity.

Solving the three equations with variables *a*, *B*, and $V_{i,t}$, the value formula is the following:

$$V_{i,t} = FCF_{i,t} + (1+R)^{-1} \cdot \left[ \hat{p} \cdot V_{i+1,t+1} + (1-\hat{p}) \cdot V_{i-1,t+1} \right], \quad (3)$$

where the term $\hat{p}$ is the risk-neutral (not market) probability such that replication is reached.

$$\hat{p} = \frac{(1+R) \cdot FCF_{i,t} - FCF_{i-1,t+1}}{FCF_{i+1,t+1} - FCF_{i-1,t+1}}.$$

If the free cash flow depends on another underlying asset $S$, which is a function of $S$, $FCF = f(S)$, then the risk-neutral probability is

$$\hat{p} = \frac{(1+R) \cdot S_{i,t} - S_{i-1,t+1}}{S_{i+1,t+1} - S_{i-1,t+1}}.$$

The generalised real options valuation model is a multi-mode (multi-switching model) allowing switching between more than two modes. The recurrent equation is the following:

$$V_{i,t} = \max_{q \in Q} \left\{ \begin{aligned} & C_{m,q} + FCF_{i,t}^{q} + \\ & (1+R)^{-1} \cdot \left[ \hat{p} \cdot V_{i+1,t+1} + (1-\hat{p}) \cdot V_{i-1,t+1} \right] \end{aligned} \right\}. \quad (4)$$

Here, $q$ is a particular mode, $Q$ is a mode set, and $m$ is an initial mode. $C_{m,q}$ is the switching cost between modes, for which a negative value means a cost and a positive value means revenue.

For more information about the real options topic, see, for example, Trigeorgis (1998).

## 2. Game theory apparatus

The important aspect of valuation is interaction. It means that a particular subject's decision depends on the decisions of other intelligent subjects and vice versa. This topic is the subject of game theory, which could be considered to be a generalised decision-making theory. Basic references are, for example, Dlouhý and Fiala (2009), Maňas (1974), Peters (2015), and Tadelis (2013).

Games are categorised according to various criteria: the number of players (two, more than two), the number of strategies (finite – discrete, infinite – continual), the cooperation type (cooperative, non-cooperative), synchronisation (simultaneous, sequential), time (static, dynamic), solution result strategies (pure, mixed), information (full, part), and the game formulation (strategic – normal, extensive).

The crucial term of game theory is equilibrium, that is, player strategies searching for equilibrium. The basic principle is the Nash equilibrium; in other words, equilibrium player strategies represent the best responses of particular players to other players' strategies. Alternatively, if equilibrium exists, if any players divert from the equilibrium strategy, they are damaged (achieve less utility).

Usually, a perfect market is supposed in a valuation; all the participants are price takers. This does not often sufficiently reflect reality, and the market structure must respond to it. It is necessary to consider and perform a valuation method in coincidence with a market structure; otherwise, the valuation cannot be correct and habitual assets are undervalued. When valuing companies and projects, games have to be categorised due to market structures: a perfect market (considerable player numbers), oligopoly (finite player numbers), duopoly (two players), and monopoly (one player).

For the Cournot model, the crucial variable is the production quantity and simultaneous decisions of players. Another possibility is a competition by price, as described by the Bertrand model. In the case of sequential decisions, the market consists of leaders and followers. The Stackelberg model represents this situation.

Duopoly is the market structure of two companies. Here, the strategy choice of the first company influence the strategy selection of the second company and vice versa, so mutual interactions are respected. In the Cournot production duopoly, the goal is profit maximisation ($z_1$, $z_2$) and the strategic decision about production quantity $(Q_1, Q_2)$ concerns given positive intervals. The inverse demand curve provides the price.

Sales $(T_1, T_2)$ and costs $(N_1, N_2)$ are expressed through a linear function; $v_1, v_2$ are unit variable costs. The price is formulated as an inverse linear demand curve function, $P = a - b \cdot (Q_1 + Q_2)$. The profit is stated as follows:

$z_1 = T_1 - N_1 = P \cdot Q_1 - v_1 \cdot Q_1$, substituting for a price,

$$\begin{aligned} z_1 &= \left[ a - b \cdot (Q_1 + Q_2) \right] \cdot Q_1 - v_1 \cdot Q_1 = \\ &(a - v_1) \cdot Q_1 - b \cdot Q_1^2 - b \cdot Q_1 \cdot Q_2 \end{aligned}.$$

The maximal profit is an extremal value as follows: $\frac{\partial z_1}{\partial Q_1} = (a - v_1) - 2b \cdot Q_1 - b \cdot Q_2 = 0$ .

It implies $Q_1 = \frac{(a - v_1) - b \cdot Q_2}{2b}$; analogically, $Q_2 = \frac{(a - v_2) - b \cdot Q_1}{2b}$. Both functions are so-called response functions stating equilibrium production in counterparty production. By mutual substitution, the equilibrium production is $Q_1 = \frac{a - 2v_1 + v_2}{3b}$ and $Q_2 = \frac{a - 2v_2 + v_1}{3b}$. The total production is the following: $Q = Q_1 + Q_2 = \frac{2a - (v_1 + v_2)}{3b}$. Substituting the total production into the equilibrium price equation, the price is $P = \frac{a + (v_1 + v_2)}{3}$. The last step is to state the profit with knowledge of production, $z_1 = \frac{\left(a - 2v_1 + v_2\right)^2}{9b}$, $z_2 = \frac{\left(a - 2v_2 + v_1\right)^2}{9b}$. The ratio $Q < \frac{a}{b}$ has to be positive to give the problem an economic rationale.

Similarly, equations for other market structures (oligopoly, monopoly, and perfect market) can be obtained. The results are presented in Table 3–1. The unit variable cost and many companies are supposed for the perfect market.

**Table 3–1** Production, price, and profit due to the market structure

| **Market structure** | **Production of the company i** $Q_i$ | **Total production** $Q$ | **Price** $P$ | **Profit of the company i** $z_i$ |
|---|---|---|---|---|
| **Monopoly** | $\frac{a - v_i}{2b}$ | $\frac{a - v_i}{2b}$ | $\frac{a + v_i}{2}$ | $\frac{\left(a - v_i\right)^2}{4b}$ |
| **Cournot duopoly** | $\frac{a - 2v_i + v_j}{3b}$ | $\frac{2a - \left(v_i + v_j\right)}{3b}$ | $\frac{a + (v_1 + v_2)}{3}$ | $\frac{\left(a - 2v_i + v_j\right)^2}{9b}$ |
| **Cournot oligopoly** | $\frac{n}{n+1} \frac{a - n \cdot v_i + \left(n-1\right)\overline{v}_{-i}}{b}$ | $\frac{n}{n+1}\left(\frac{a - \overline{v}}{b}\right)$ | $\frac{a + n \cdot \overline{v}}{n+1}$ | $\frac{1}{b}\left(\frac{a - n \cdot v_i + \left(n-1\right)\overline{v}_{-i}}{n+1}\right)^2$ |
| **Perfect market** | $\frac{1}{n+1} \frac{a - v}{b}$ | $\frac{n}{n+1}\left(\frac{a - v}{b}\right)$ | $\frac{a + n \cdot v}{n+1}$ | $\frac{1}{b}\left(\frac{a - v}{n+1}\right)^2$ |

Legend: $\overline{v}_{-i} = \sum_{j \neq i}^{n-1} v_j$ , $\overline{v} = \sum_{j}^{n} v_j$

A bimatrix game signifies a two-player game with discrete strategies. The matrix of each player presents payoffs for all combinations of strategies. Commonly, the game is a non-zero-sum one. The primary objective of the players' choice of strategy is to find an equilibrium strategy. The solution can involve pure strategies or mixed strategies. Firstly, every player selects only one strategy. The methods of *best responses* and *iterative elimination non-dominated strategies* can be applied. Secondly, the players select strategies with probability. The optimisation mixed problem is used.

The player searches for the best responses (maximal value) to counter-player strategies by applying the best response method. The best responses are shaded in Figure 3–1. For player A, if player B selects strategy B1, the best reaction is A1; if the player selects B2, the best response is A1. The reaction of player B is the following: if player A chooses A1, B selects B2; for choice A1, player B selects B1. The equilibrium point presents the combination of A1 and B2 with a payoff of 500 for A and 700 for B.

| Payoff matrix Player A | | Player B | |
|---|---|---|---|
| | | Strategy B1 | Strategy B2 |
| Player A | Strategy A1 | 600 | 500 |
| | Strategy A2 | 300 | 400 |

| Payoff matrix Player B | | Player B | |
|---|---|---|---|
| | | Strategy B1 | Strategy B2 |
| Player A | Strategy A1 | 500 | 700 |
| | Strategy A2 | 300 | 200 |

**Figure 3–1** Best response equilibria in a bimatrix game

In pure strategies, except for one unique equilibrium, there can be more or no solutions. In the case of more solutions, the most stable solution is sought, and the concept of the so-called minimisation of shaken hands is used. The common solution is in mixed strategies, encompassing pure strategies as a subset. The formulation of the non-linear optimisation problem is reported, for example, by Dlouhý and Fiala (2009), Maňas (1974), and Peters (2015) as the following:

**Problem I (bimatrix game)**

$$\max_{p_i,q_j,v,w} \sum_i^M \sum_j^N p_i a_{ij} q_j + \sum_i^M \sum_j^N p_i b_{ij} q_j - v - w$$

$$\sum_j^N a_{ij} \cdot q_j \le v, \quad \forall i$$

$$\sum_i^M b_{ij} \cdot p_i \le w, \quad \forall j$$

Here, $p_i$ and $q_j$ are the strategy probabilities of player A and, respectively, player B, $a_{ij}$ and $b_{ij}$ are the payoffs of player A and, respectively, player B, $v$ and $w$ are variables, $i$ and $j$ are the indexes of the strategies of player A and, respectively, player B, and $M$ and $N$ are the number of strategies of player A and, respectively, player B.

Problem I can be modified into a more suitable computation formulation. After dividing the equations by $v$ and $w$ and substituting $x_i = p_i / v$ and $x_i = p_i / w$, Problem II is as follows:

**Problem II (modified bimatrix game)**

$$\max_{p,q,a,b} \sum_i^M \sum_i^N x_i a_{ij} y_j + \sum_i^M \sum_i^N x_i b_{ij} y_j - 1 - 1$$

$$\sum_j^N a_{ij} \cdot y_j \le 1, \quad \forall i$$

$$\sum_i^M b_{ij} \cdot x_i \le 1, \quad \forall j$$

## 3. Real game options valuation model

The payoff function depends on the underlying asset random process and independently on interactions during the valuation in the real options method. In this section, the approach is generalised. It is supposed that a payoff function is determined, except for the underlying random asset movement, by mutual interactions, so it depends on other players' decision (choice of strategies). The valuation of the game real options is similar to the real options valuation, except the payoff function is only given by game theory with interactions. The topic in discrete time is worked out for example by Chevalier-Roignant (2011) and Smit and Trigeorgis (2004, 2017).

Valuation considering the actions of other companies is a generalised approach including risk, flexibility, and interaction. Game theory instruments serve to model interactivity. The crucial term of game theory is equilibrium. The term mode is substituted by the term strategy in comparing the real options model. The generalised multi-mode real options with interaction model is modified to a generalised multi-strategy model as follows:

$$V_{i,t} = \left\{ \begin{matrix} C_{n,w_k} + FCF_{i,t}^{w_k,w_{-k}} + \\ (1+R)^{-1} \cdot \left[ \hat{p} \cdot V_{i+1,t+1} + (1-\hat{p}) \cdot V_{i-1,t+1} \right] \end{matrix} \right\}, \quad (5)$$

where, for the k[th] player, $w_k \in W^k$ (scalar or vector) is the equilibrium strategy and $w_{-k} \in W^{-k}$ (vector or matrix) is the strategy set, for other players, $w_{-k}$ (vector or matrix) is the equilibrium strategy and $W^{-k}$ (matrix) is the strategy set, and $C_{n,w_k}$ (scalar or vector) is the switching cost between strategies.

The binomial model with risk-neutral valuation, the two-phase method, and the game payoff function is presented.

**Valuation procedure of the real game options model**

**(i) Determination of the underlying asset (factor) random process**

An approach based on an expert's estimation or random process calibration (e.g., Browns, CIR, Ho-Lee).

**(ii) Equilibrium game payoff determination**

Equilibrium payoffs are calculated due to the game model.

**(iii) Stating terminal (continuum) value**

Terminal value calculation (e.g. perpetuity and growing perpetuity) due to states $V_{i,T}$.

**(iv) Determination value for particular states**

Backward induction procedure due to (5):

$$V_{i,t} = \left\{ \begin{matrix} C_{n,w_k} + FCF_{i,t}^{w_k} + \\ (1+R)^{-1} \cdot \left[ \hat{p} \cdot V_{i+1,t+1} + (1-\hat{p}) \cdot V_{i-1,t+1} \right] \end{matrix} \right\}$$

**(v) Determination value of an option**

The value of the real game option at the beginning $V_0$.

## 4. Company valuation in a duopoly market structure under random demand (simplified example)

The objective is to state the value of the companies (company A and company B) operating in a production duopoly. We simplistically assume that depreciation equals the investment, other fixed costs are not considered, the net working capital change is zero, and the transaction cost and taxes are neglected. Therefore, the free cash flow is identical to the profit. A two-phase discounted cash flow valuation method is applied, and a non-cooperative Cournot production duopoly determines the equilibrium profit. An inverse linear demand curve gives the production price. Random development is given by parameter $a$ of the demand curve, which obeys a geometric Brownian process. For the calculation, a binomial model is used.

With input parameter value $a$ = 10, the companies' unit variable costs are $v_A$ = 2, and $v_B$ = 3. Hence, company A is more effective than company B, the cost of capital for both companies $R_{AB}$ = 20%, and the risk-free rate $R$ = 10%. The development of parameter $a$ is apparent from the binomial model (Figure 5–1), and the input parameters, including the calculated up-index and down-index, along with the risk-neutral probabilities, are shown in Table 5–1.

| state \ time | 0 | 1 | 2 |
|---|---|---|---|
| | | | *uu* |
| 2 | | *u* | 15,63 |
| 1 | | 12,50 | *ud* = *du* |
| 0 | 10,00 | *d* | 10,00 |
| -1 | | 8,00 | *dd* |
| -2 | | | 6,40 |

**Figure 5–1** Development of parameter $a$ (binomial model)

**Table 5–1** Input and calculated parameters

| Item | Parameter | Value | Parameter | Value |
|---|---|---|---|---|
| **Price** | $a$ | 10 | $b$ | 0,5 |
| **Unit variable cost** | $v_A$ | 2 | $v_B$ | 3 |
| **Rates** | $R$ | 0,10 | $R_{AB}$ | 0,2 |
| **Indices** | $U$ | 1,25 | $D$ | 0,8 |
| **Probabilities** | $\hat{p}$ | 0,67 | $1-\hat{p}$ | 0,33 |

Valuations applying the two-phase method, binomial model, replication approach, American options, and two payoff matrices are implemented, and a pure strategy is supposed. The recurrent valuation equation formula is as follows:

$$V^A_{i,t} = \left\{ \begin{matrix} z_{i,t}^{w_A, w_B} + \\ (1+R)^{-1} \cdot \left[ \hat{p} \cdot V^A_{i+1,t+1} + (1-\hat{p}) \cdot V^A_{i-1,t+1} \right] \end{matrix} \right\}, \quad (6)$$

$$V^B_{i,t} = \left\{ \begin{matrix} z_{i,t}^{w_B, w_A} + \\ (1+R)^{-1} \cdot \left[ \hat{p} \cdot V^B_{i+1,t+1} + (1-\hat{p}) \cdot V^B_{i-1,t+1} \right] \end{matrix} \right\}, \quad (7)$$

where $V^A_{i,t}$ and $V^B_{i,t}$ are values, $z^A_{i,t}$ and $z^B_{i,t}$ are profit, $w_A \in W^A$ and $w_B \in W^B$ (scalar) are the equilibrium strategy, $W^A$ and $W^B$ are vectors of strategies, $\hat{p}$ is the risk-neutral probability, and $R$ is the risk-free rate.

Duopoly companies' equilibrium profit of concrete nodes, shown in Table 5–1, are $z = \frac{(a-2v+v)}{9b}$ and $z = \frac{(a-2v+v)}{9b}$. The backward induction procedure, with the first value at a terminal time using perpetuity, is calculated as $V = \frac{z}{R_{AB}}$, then the node values are calculated using (6) and (7). Table 5–1 presents the results for nodes, including price $P = \frac{a+(v_A+v_B)}{3}$ and production $Q_A = \frac{a-2v_A+v_B}{3b}$, $Q_B = \frac{a-2v_B+v_A}{3b}$. The development equilibrium profit and company value are shown in Figure 5–1.

**Table 5–2** Calculated values, process, production, and profit of companies A and B

| **State** | | **beginning** | | ***u*** | | ***d*** | | ***uu*** | | ***ud = du*** | | ***dd*** | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **company** | | **A** | **B** | **A** | **B** | **A** | **B** | **A** | **B** | **A** | **B** | **A** | **B** |
| **coef.** | ***a*** | 10,00 | | 12,50 | | 8,00 | | 15,63 | | 10,00 | | 6,40 | |
| **price** | ***P*** | 5,00 | | 5,83 | | 4,33 | | 6,88 | | 5,00 | | 3,80 | |
| **production** | ***Q*** | 1,50 | 1,00 | 1,92 | 1,42 | 1,17 | 0,67 | 2,44 | 1,94 | 1,50 | 1,00 | 0,90 | 0,40 |
| **profit** | ***z*** | 18,00 | 8,00 | 29,39 | 16,06 | 10,89 | 3,56 | 47,53 | 30,03 | 18,00 | 8,00 | 6,48 | 1,28 |

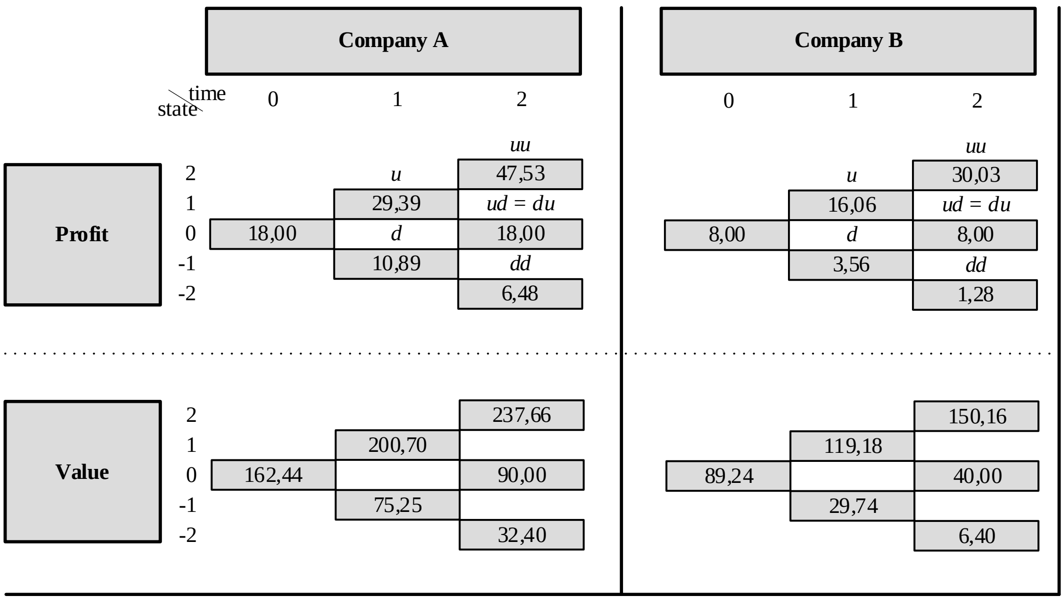


**Figure 5–2** Equilibrium profit and value development of company A and company B

It is evident that company A's value is 162,44 m. u., company B's value is 89.24 m. u., and the impact of the companies' effectiveness is apparent. The computed values reflect the production duopoly conditions and the economic level of the particular companies. According to states and time, the problem allows the analysis of the valuation circumstances and equilibrium parameters; see Table 5–1, which presents the prices $P$, companies' production $Q_A$ and $Q_B$, and profit $z_A$ and $z_A$. It is easy to show that company values in a duopoly market could be computed comparably under an oligopoly market structure by applying the formulas of Table 3–1.

## 5. Conclusion

The real options approach could be considered as a company valuation concept that reflects uncertainty and flexibility. An essential element of the valuation environment is interaction. This phenomenon, embodying the mutual relationships among companies, is dealt with using game theory. However, the introduced aspect is often neglected even if it substantially influences the company value under specific non-perfect market structures. Hence, the real game options method encompasses this phenomenon.

The methodology of the game real options valuation model, based on a two-phase method in discrete time, was developed and formulated and an illustrative example was presented in the paper. The computation procedure of real game options was described. Games with non-perfect market structures were formulated, specifically duopoly and oligopoly Cournot production games. The duopoly market structure was implemented and calculated in the illustrative example.

It was found that two-phase real game options valuation in discrete time is a suitable valuation approach for companies reflecting non-perfect market structures.

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