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Break-even analysis under randomness with heavy-tailed distribution

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Abstract

Break-even analysis is a tool suitable for making short-term decisions about the quantity of production. Traditional break-even analysis is based on certain assumptions among which the most important are the following limitations: variable costs are linearly dependent on sales volume; price of the product is stable; fixed costs do not change. Moreover, we assume that all the input variables (variable costs per unit, fixed costs and price of the product) are known with certainty. However, these variables may be random and thus not known in advance. For instance, a firm can be price-taker – the price of the product is a random variable determined by the market, variable costs per unit depend on the price of raw materials, which again cannot be known in advance with certainty. In our paper, we discuss the break-even analysis introducing randomness. We focus on two input variables – the price of the product, which influences the revenues, and the variable costs per unit, which influence the costs. Both random inputs are supposed to follow joint normal distribution and normal inverse Gaussian distributions joined together by copula function.

Keywords

break-even analysis; copula function; NIG

JEL Classification: G31, M21

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1. Introduction

The traditional break-even analysis is suitable tool for making short-term decisions about the quantity of production. It is based on the fundamental limitations and assumptions among which we can mention the following: all production is realized (sold); revenues and total costs vary only due to changes in sales volume; all costs may be divided into fixed and variable costs; variable and total costs evolve linearly; fixed costs do not change; the price of the product remains unchanged; technology and organization of production does not change; production process is continuous. These assumptions imply that the revenues and the total costs can be expressed as linear equations, or lines in the case of graphical representation – for detailed explanation see e.g. Cafferky and Wentworth (2014) or Warren et al. (2015).

Under such a setting, we can calculate the break-even point, i.e. sales volume at which total revenues are equal to total costs, from the input variables, which are the price of the product, variable costs per unit of product and fixed costs. In traditional break-even analysis, all these input variables are assumed to be known with certainty, which clearly is the case of the fixed costs (usually repetitive costs that are known in advance). However, even assuming a short-term period, the price of the product and variable costs may be known only with uncertainty, i.e. we do not know the exact value, but at least we know its probability distribution. An example can be the firm selling all its production in foreign currency, thus, the revenues (in domestic currency) depend on the foreign exchange rate. Another example is the firm manufacturing a product the price of which is defined by the market – imagine, for example, a company producing electricity and selling it on a spot market. On the other hand, the variable costs per unit of product are mostly raw materials, the prices of which do not have to be known in advance.

In the literature, there are two approaches of introducing uncertainty into traditional break-even analysis: stochastic randomness and fuzzy uncertainty. For the first approach we can mention, for example, papers of Jaedicke and Robichek (1964), who assumed profits to be a random variable, followed by Dickinson (1974) and Yunker and Yunker (2003) who assumed the price to be neither a constant nor a random variable but rather the firm's basic decision variable. In these papers, the attention is focused mostly on the situation with single source of randomness, which is modelled by normal (Gaussian) distribution. The examples of the later approach are papers of Yuan (2009) or Chrysafis and Papadopoulos (2009).

The goal of the paper is to modify the traditional break-even-point analysis for the assumption of random input variables and compare the results of both approaches for the practical example. In the paper, we also study the influence of distribution shape on the results; specifically, we compare normal distribution already studied by Kresta and Lisztwanová (2017) with heavy-tailed normal inverse Gaussian distribution.

The structure of the paper is as follows. In the next section, we briefly recap the traditional break-even analysis. In the third section we introduce the break-even analysis with random variables. The practical example is provided in the fourth section. Finally, the fifth section is the conclusion.

2. Break-even analysis under certainty

The break-even point represents a sales volume at which total revenues (*TR*) are equal to total costs (*TC*) and at which neither profit nor loss is made,

$$TR = TC \ . \tag{1}$$

Total revenues can be computed as the price of the product (p) multiplied by sales volume (Q) and total costs can be divided into fixed costs (*FC*) and variable costs (*VC*). Variable costs are computed as variable costs per unit (*vc*) multiplied by sales volume (Q). After substitutions, we get the following equations,

$$p \cdot Q = FC + vc \cdot Q \ , \tag{2}$$

which we solve for Q,

$$Q = \frac{FC}{p - vc} \ . \tag{3}$$

Moreover, we can substitute the term $p - vc$ by m (unit margin),

$$Q = \frac{FC}{m} \ . \tag{4}$$

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3. Break-even analysis under randomness

Assume that the price of the product and variable costs per unit are not known with certainty, but can be described by proper probability distribution. In this case, we cannot calculate the break-even point for which we know for sure that there will be no loss, but we must specify the confidence level, i.e. the probability with which there will be no loss. For example, if we assume a confidence level of 85%, the calculated break-even point under randomness means that for this sales volume there is an 85% probability that there will be no loss. On the other hand, there is a 15% chance that there will be a loss.

Based on the applied probability distributions of random inputs and their dependence structure we can distinguish the simple case of joint normal distribution (solvable by analytical formula) and a more general case for which Monte Carlo simulation must be applied.

3.1 Joint normal distribution – analytical formula

If $p \sim N(\mu_p, \sigma_p)$ and $vc \sim N(\mu_{vc}, \sigma_{vc})$ and *p* and *v* are correlated with correlation coefficient $\rho_{p,vc}$ then $m \sim N(\mu_p - \mu_{vc}, \sqrt{\sigma_p^2 + \sigma_{vc}^2 - 2 \cdot \sigma_p \cdot \sigma_{vc} \cdot \rho_{p,vc}})$.
Assuming a confidence level α, we can calculate the break-even point under randomness similarly to equation (4) as follows,

$$Q = \frac{FC}{\Phi^{-1}\left(1-\alpha; \mu_p - \mu_{vc}; \sqrt{\sigma_p^2 + \sigma_{vc}^2 - 2 \cdot \sigma_p \cdot \sigma_{vc} \cdot \rho_{p,vc}}\right)} . \quad (5)$$

3.2 Simulation approach

In formula (5) we assumed the simplifying example of joint normal distribution, which does not have to be the case in real-world applications. Assume, for example, that the product is sold in foreign currency, which influences the revenues (no hedging of foreign exchange rate). Under such a set-up, we should assume some heavy-tailed distribution and apply the simulation approach. We proceed as follows.

1. We simulate random variable costs per unit and prices per unit with a corresponding dependence structure and marginal distributions, see e.g. Kresta (2010);
2. for each simulation we compute the break-even point;
3. finally, we compute the quantile of simulated break-even points.

In order to simulate the random variables we shall apply the copula approach with some heavy-tailed distribution. Thus, further we will briefly introduce the copula functions and probability distributions applied in a practical example.

3.2.1 Copula functions

A useful tool for the simulation of dependent random variables are the copula functions. Further, we will explain only the basic theory of copula functions, for more detailed explanation see, for example, Nelsen (2006), Rank (2006) or Cherubini et al. (2004).

Assume two potentially dependent random variables *X, Y* with marginal distribution functions F_X and F_Y and a joint distribution function $F_{X,Y}$. Then, following Sklar's theorem:

$$F_{X,Y}(x, y) = \mathcal{C}\left(F_X(x), F_Y(y)\right). \quad (6)$$

Formulation (6) should be understood such that the joint distribution function gives us two distinct pieces of information: (i) the marginal distributions of the random variables; and (ii) the dependency function of the distributions. Hence, while the former is given by F_X and F_Y, the copula function specifies the dependency. Only when we put the two pieces of information together, we have sufficient knowledge about the pair of random variables *X, Y*.

With some simplification, we can distinguish copulas in the form of elliptical distributions and copulas from the Archimedean family. The main difference between these two forms lies in the methods of construction and estimation. While for the latter the primary assumption is to define the generator function, for the former the knowledge of the related joint distribution function (e.g. Gaussian, Student) is sufficient.

3.2.2 Gaussian distribution

The Gaussian (also called normal) distribution is well-known continuous probability distribution, which can be easily recognized by its bell curve shape. It can be characterized by the following probability density function,

$$f_N\left(x; \mu, \sigma\right) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} , \quad (7)$$

and cumulative distribution function,

$$\Phi\left(x; \mu, \sigma\right) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt , \quad (8)$$

where μ and σ are the parameters that determine the shape of the distribution. The first parameter defines the location (mean, median and mode of the distribution are equal to this parameter) and the second parameter defines the scale of the distribution (standard deviation is equal to this parameter). Normal distribution is symmetrical.

3.2.3 Normal inverse Gaussian distribution

Normal inverse Gaussian distribution (hencefort NIG) was defined in Barndorff–Nielsen (1995). Assuming parameters $\alpha > 0$, $-\alpha < \beta < \alpha$, $\delta > 0$ and μ, the distribution has the following probability density function,

$$f_{NIG}\left(x;\mu,\alpha,\beta,\delta\right) = \frac{\alpha\delta}{\pi} \exp\left(\delta\sqrt{\alpha^2-\beta^2}+\beta\left(x-\mu\right)\right) \frac{K_1\left(\alpha\sqrt{\delta^2+\left(x-\mu\right)^2}\right)}{\sqrt{\delta^2+\left(x-\mu\right)^2}}, \quad (9)$$

and corresponding cumulative distribution function,

$$F_{NIG}\left(x;\mu,\alpha,\beta,\delta\right) = \frac{\alpha\delta}{\pi} \int_{-\infty}^{x} \left[\frac{K_1\left(\alpha\sqrt{\delta^2+\left(t-\mu\right)^2}\right)}{\sqrt{\delta^2+\left(t-\mu\right)^2}} \exp\left(\delta\sqrt{\alpha^2-\beta^2}+\beta\left(t-\mu\right)\right) \right] dt, \quad (10)$$

where $K_\lambda(x)$ denotes modified Bessel function of the third kind. In this distribution, parameter μ influences the location, α influences the tail heaviness, β influences the asymmetry and δ influences the scale of probability distribution.

The first four central moments of both distributions described above are summarized in Table 1. As is clear, skewness and kurtosis of Gaussian distribution cannot be influenced – skewness is always zero (i.e. Gaussian distribution is symmetrical) and kurtosis is always equal to 3. The formulas in Table 1 can be applied for the estimation of parameters by means of method of moments (see e.g. Tichý, 2011).

4. Practical example

In this section, we provide a practical example in which we assume both all input variables are certain and two input variables are random. Under randomness, we assume two probability distributions, namely normal (Gaussian) distribution and NIG distribution. In total, we study four different cases, which differ in the specification of product price and variable costs per unit, see Table 2. In the first specification, we assume that the price is 100 and variable costs per unit are 60 with certainty. In the second specification, we assume the price to be normally distributed with 95% probability of being between 80 and 120 and variable costs to be normally distributed with 95% probability of being between 50 and 70. In the third and fourth specification, we assume price and variable costs to have NIG distribution with parameters such that the location and scale are the same as in the previous specification but the tails are heavier than those of Gaussian (third and fourth case) and distributions are skewed (fourth case). It is important to ensure the same scale of the distributions as this influences the results. If we increase the scale (standard deviation) of the distributions, i.e. we are less certain about the random input values, we obtain higher break-even volumes for the same confidence level. Thus, in order to obtain comparable results, the standard deviation is the same for all random specifications; these only differ in tail heaviness and skewness.

Moreover, we assume linear dependence modelled by the Gaussian copula function with a correlation parameter of 0.5 in all specifications as well as fixed costs to be 1,000,000.

Table 1 Characteristics of the distributions

	$N\left(\mu,\sigma\right)$	$NIG\left(\mu,\alpha,\beta,\delta\right)$
Mean	μ	$\mu+\delta\beta\left(\alpha^2-\beta^2\right)^{-\frac{1}{2}}$
Standard deviation	σ	$\alpha^2\delta\left(\alpha^2-\beta^2\right)^{-\frac{3}{2}}$
Skewness	0	$3\beta\alpha^{-1}\delta^{-\frac{1}{2}}\left(\alpha^2-\beta^2\right)^{-\frac{1}{4}}$
Kurtosis	3	$3\left(1+\frac{\alpha^2+4\beta^2}{\delta\alpha^2\sqrt{\alpha^2-\beta^2}}\right)$

Table 2 Input variables

Item	Certainty	Randomness – Gaussian	Randomness – symmetrical NIG	Randomness – skewed NIG
Product price (*p*)	100	N(100;10)	NIG(100;0.0612;0;6.1237)	NIG(103;0.0911;–0.0353;7.141)
Variable costs per unit (*vc*)	60	N(60;5)	NIG(60;0.1225;0;3.0619)	NIG(60.33;0.1265;–0.0132;3.111)
Fixed costs (*FC*)	1,000,000			

4.1 Break-even point under certainty

By entering the values of input variables into equation (3) we obtain the following break-even point under certainty:

$$Q = \frac{1,000,000}{100-60} = 25,000\,. \tag{11}$$

We can conclude that in order to avoid the loss we have to produce and sell at least 25,000 pieces of product.

4.2 Analytical computation

By entering the values of input variables into equation (5) we obtain the following break-even point under randomness:

$$\begin{aligned} Q &= \frac{1,000,000}{\Phi^{-1}\left(1-0.85;100-60;\sqrt{10^2+5^2-2\cdot 10\cdot 5\cdot 0.5}\right)} \\ &= \frac{1,000,000}{\Phi^{-1}\left(0.15;40;8.66\right)} \\ &= 32,233. \end{aligned} \tag{12}$$

We can conclude that in order to be 85% sure that the loss will be avoided, the company have to produce and sell at least 32,233 pieces of product. For such a quantity of production, we can calculate the expected profit $E(P)$ as follows,

$$E(P) = E(p - vc)\cdot Q - FC\,, \tag{13}$$

$$E(P) = 40\cdot 32,233 - 1,000,000 = 289,315\,. \tag{14}$$

From equation (5), it is obvious that the value of the break-even quantity and expected profit depends on the confidence level. Thus, we perform the sensitivity analysis in Table 3. It can be noted that for a confidence level of 50% we obtain the same results as in the case of break-even analysis under certainty. This is because in this case we are calculating mean value (median and mean is the same for joint normal distribution), i.e. it is enough to work with the mean values of the distributions of both random variables, which are the same as in the case of certainty. The next observation is that the higher the confidence level, the higher the break-even point, which corresponds with a higher expected profit. This is logical, as the more confident we want to be that there will be no loss, the further we get into the tail of probability distribution. Simply speaking, the more confident we want to be that there will be no loss, the more we have to produce – the higher production volume can balance a possibly unfavourable low unit margin *m*.

4.3 Simulation approach

In this section we perform simulation approach in order to quantify the break-even point under randomness. We examine two distributions: normal distribution and NIG distribution, for their parameters see Table 2. The procedure of simulation and break-even point quantification is described in section 3.2. The calculations are performed in MATLAB and the source code is attached in the appendix. The number of simulated trials is set to 5,000,000 for both distributions, which guarantees that the results become stable and do not change significantly when recalculated under a different set-up of the pseudorandom numbers generator.

4.3.1 Break-even point under Gaussian distribution

The results, i.e. the break-even point under randomness and corresponding expected profit, under the assumption of joint normal distribution are presented in Table 4. As can be seen, the results are close to the results obtained from the analytical formula. We do not comment on the obtained results again and the reader is referenced to comments in section 4.2.

Table 3 Sensitivity of break-even point to specification of the confidence level

Confidence level	50%	75%	85%	90%	95%
Break-even point	25,000	29,275	32,233	34,600	38,827
Expected profit	0	171,003	289,315	384,014	553,087

Table 4 Break-even points under randomness modelled by joint normal distribution

Confidence level	50%	75%	85%	90%	95%
Break-even point	24,996	29,278	32,226	34,591	38,827
Expected profit	-148	171,138	289,053	383,637	553,070

4.3.2 Break-even point under NIG distribution

In this section we assume the heavy-tailed and skewed distributions. Starting from the previous joint normal distribution, we first increase the heaviness of the tails and next we make the distribution skewed.

Thus, we first assume symmetrical NIG distributions with excess kurtosis equal to 8. The means and standard deviations are the same as in the case of joint normal distribution (see the parameters in Table 2).

The results are presented in Table 5. As can be seen, for a confidence level of 50% we get the same break-even point as in the case of certainty (the joint distribution is still symmetrical with means/medians equal to the values under certainty). Higher confidence levels mean higher break-even volumes – the rationale for this has already been discussed, however the break-even volumes are lower than in the case of normal distribution. One may think that the results are incorrect as we get lower values for tail-heavier distribution, but the opposite is true. We must think about what the tail is and what the central part of the distribution is and how they are connected. The tail of distribution is only a small part of the probability distribution at the edges. The rest will be called the central part. Then, if the tails are heavy (i.e. the probability of these values is higher, let's say, compared to normal distribution) then the central part must be *less heavy*, i.e. values in this part have lower (cumulative) probability and vice versa. So we can see, that the higher the heaviness of the tail the higher the break-even volumes in the tail and the lower the break-even points in the central part. The assumed confidence levels are mostly in the central part of the distribution. As 95% can be considered as the tail, we can see that the break-even volume started to accelerate the increase there. Actually, if we analyse even higher confidence levels – see Figure 1 – we can conclude that break-even volumes are higher under NIG distribution only for confidence levels of 96.5% and higher. As can be seen, after this point the increase of break-even volume starts to accelerate. However, in our opinion, these values of confidence level are not applicable in the real world. As can be seen, under these experimental settings, even considering a 95% confidence level, the result is to produce and sell 37,374 pieces of product, for which we can expect a profit of 494,959, which is actually half of the value of fixed costs.

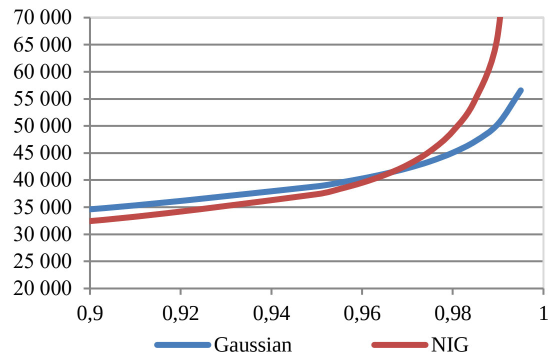

Figure 1 Comparison of break-even volumes for joint normal and symmetrical NIG distributions

Moreover, we study the effect of the skewness on the results. We assume the left-skewed distributions with excess kurtosis of 8, for NIG distribution parameters see Table 2. The results of the simulation are presented in Table 6. As can be seen, the values differ significantly only for high confidence levels (90% and 95%). For lower confidence levels the results are similar to those of symmetrical specification (see Table 5).

5. Conclusion

The break-even analysis is a traditional tool for making short-term decisions about the quantity of production. In this paper, we introduce randomness into the analysis. The contribution of the paper is twofold: we modify the break-even point analysis for the assumption of random input variables and compare the results of both approaches for the practical example.

Table 5 Break-even points under randomness modelled by symmetrical NIG distribution

Confidence level	50%	75%	85%	90%	95%
Break-even point	24,998	27,971	30,277	32,423	37,374
Expected profit	-74	118,838	211,062	296,918	494,959

Table 6 Break-even points under randomness modelled by skewed NIG distribution

Confidence level	50%	75%	85%	90%	95%
Break-even point	24,450	27,655	30,388	33,089	39,839
Expected profit	-22,007	106,195	215,515	323,579	593,559

Moreover, we study the influence of distribution shape on the results.

Based on the results, we can summarize the following findings. Break-even volume is generally higher under randomness and depends (among others) on the confidence level assumed, i.e. the more certain of no loss we want to be, the higher the break-even volume needed. We can also expect that the break-even volume increases with increased uncertainty (the scale of distributions); however, we have not examined this in the paper. Moreover, we have found out that the more tail-heavy the distribution we assume, the lower is the calculated break-even volume. That is because the confidence levels we assumed in the paper are low compared, for example, to the modelling of risks in financial institutions. However, we think that for managerial decisions, the lower values of confidence levels are more beneficial as the input variables are only roughly estimated, and for high confidence levels, we are mooving away from what we can expect on average – actually, we can expect a high positive profit. The skewness has small effect on the break-even volumes.

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Appendix

Code 1 Calculations in Matlab

```
Ntrials=5000000;
alphas=[0.5 0.75 0.85 0.9 0.95];

%% BE under certainty
p=100;
vc=60;
FC=1000000;
display('BE under certainty');
Q=FC/(p-vc)

%% BE under randomness - analytical solution
p_sigma=10;
vc_sigma=5;
rho=0.5;
display('BE under randomness - analytical solution\n');
analytical_quantiles=FC./norminv(1-alphas,p-vc,sqrt(p_sigma^2+vc_sigma^2-
2*p_sigma*vc_sigma*rho))
analytical_ep=a_quantiles*(p-vc)-FC;

%% BE under randomness - simulation approach
U=copularnd('Gaussian',rho,Ntrials);
g_p=icdf('Normal',U(:,1),p,p_sigma);
g_vc=icdf('Normal',U(:,2),vc,vc_sigma);
m=g_p-g_vc;
m(m<0)=0;
Q=FC./m;
display('BE under randomness - Gaussian\n');
gaussian_quantiles=quantile(Q,alphas)
gaussian_ep=g_quantiles*(p-vc)-FC;

% NIG distribution
% NIG distribution toolbox can be downloaded at
% https://www.mathworks.com/matlabcentral/fileexchange/
% 10934-normal-inverse-gaussian--nig--distribution-updated-version

%nig_p=niginv(U(:,1),0.0612,0,100,6.1237);
%nig_vc=niginv(U(:,2),0.1225,0,60,3.0619);
nig_p=niginv(U(:,1),0.0911,-0.0353,103,7.1414);
nig_vc=niginv(U(:,2),0.1265,-0.0132,60.3261,3.1107);
nig_p(isnan(nig_p))=p;
nig_vc(isnan(nig_vc))=vc;
m=nig_p-nig_vc;
m(m<0)=0;
Q=FC./m;
display('BE under randomness - NIG\n');
nig_quantiles=quantile(Q,alphas)
nig_ep=nig_quantiles*(p-vc)-FC;
```